Eudoxus

Erin Sondgeroth
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There were many brilliant mathematicians from the time period of 500 BC until 300 BC, including Pythagoras and Euclid. Another fundamental figure from that era was Greek mathematician Eudoxus of Cnidus. Eudoxus was born in Cnidus (present-day Turkey) in approximately 408 BC and died in 355 BC at the age of 53. Eudoxus studied math under Archytus in Tarentum, medicine under Philistium in Sicily, astronomy in Egytpt, and philosophy and rhetoric under Plato in Athens. After his many years of studying, Eudoxus established his own school at Cyzicus, where had many pupils. In 365 BC Eudoxus moved his school to Athens in order to work as a colleague of Plato. It was during this time that Eudoxus completed some of his best work and the reasoning for why he is considered the leading mathematician and astronomer of his day.

In the field of astronomy, Eudoxus's best-known work was his planetary model. This model had a spherical Earth that was at rest in the center of the universe and twenty-seven concentric spheres that held the fixed stars, sun, moon, and other planets that moved in orbit around the Earth. While this model has clearly been disproved, it was a prominent model for at least fifty years because it explained many astronomical phenomenon of that time, including the sunrise and sunset, the fixed constellations, and movement of other planets in orbits.

Luckily, Eudoxus's theories had much more success in the field of mathematics. Many of his proposition, theories, and proofs are common knowledge in the math world today. Some of Eudoxus's best-known mathematical contributions include his proofs for the volume of pyramids and cones, his Theory of Proportion, his propositions on magnitudes, and his Method of Exhaustion. Eudoxus was able to use calculus and integrals in order to prove that the volume of a pyramid is one-third the volume of the prism that has the same base and height as the pyramid and that the volume of a cone is one-third the volume of the cylinder that has the same base and height of the cone. In order to prove the volume of a pyramid, Eudoxus considered the total volume of stacked prisms of decreasing size. Eventually, Eudoxus argued that if the number of prisms was increased and the height of each decreased until there were many, that it was possible to calculate the volume of a pyramid using integrals.

Eudoxus's proof for the volume of a pyramid went as follows:

Theorem: The volume of a pyramid, V, is given by $V = (Abase * h)/3$, where h is the perpendicular height.

Proof: Consider any pyramid with perpendicular height,h, and area of the base, A. Now, imagine that the pyramid is split into n number of layers. Using the idea of similarity, the k th layer will have a base with dimensions k/n as a fraction of the original base. So the area of the k th base will be $(k/n)^2A$. The area of the base of each layer will be

$$(1/n)^2A, (2/n)^2A, \dots (n/n)^2A$$

By considering each layer to be a prism, each will be h/n units tall and the volume of the k th slab will be $(h/n)(k/n)^2A$. The volume can then be approximated

$$V = (h/n)(1/n)^2A + (h/n)(2/n)^2A + (h/n)(3/n)^2A + ... + (h/n)(n/n)^2A$$

which may be rewritten as:

$$(Ah/(n^3))(1^2 + 2^2 + 3^2 + ... + n^2)$$

Using the result:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

we get:

$$V = (Ah/(n^3))(1/6n)(n+1)(2n+1) = (Ah/6)(1+1/n)(2+1/n)$$

It is clear that the greater the value of n, the better the approximation. Thus, if n approaches infinity, it can be concluded that 1/n becomes less significant and is considered to be 0. Hence V tends towards $(Ah/6)(1+0)(2+0) = Ah/3$.

Q.E.D.

Eudoxus made many contributions to mathematics that are widely accepted and imperatively used today.