

Ibn al-Banna

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Ibn al-Banna's life

Ibn al-Banna was born on December 29 in the year 1256. His full name was Abu'l-Abbas Ahmad ibn Muhammad Ibn Uthman al-Azdi. He was born in Marrakech, Morocco. However, this is also speculation that he was born in Grenada, Spain and moved to Northern Africa for educational purposes. Despite this uncertainty, it can be affirmed that Ibn al-Banna spent the majority of his life in Morocco. Ibn al-Banna himself, had studied fractional numbers, geometry, and the other mathematical advances that the Muslims had made in the preceding centuries to mathematics. He specifically studied the thirteen books that made up Euclid's *Elements*, which goes into more detail on plane geometry, three dimensional geometry, number theory, and geometric and abstract algebra. He studied so many branches of mathematics that he reportedly has at least seventeen majors.

For a portion of his education, he frequented Aghmat, where he studied under the mathematician Abu Abd Allah al-Hazmiri, who passed away in 1279. The influence of this mathematician is what is thought to be the reason Ibn al-Banna became interested in astrology and astronomy, which is how he eventually gained the reputation of being a Sufi. After his own education, Ibn al-Banna began to teach in Fes, which was the then capital of Morocco. In Fes, Ibn al-Banna taught all branches of mathematics known at the time at the city's university as well as at the great mosque of Marrakech. He taught geometry, arithmetic, astronomy, and algebra.

It is thought by many historians that Ibn al-Banna became a seriously practicing astrologer under the guidance of Marinid sultan Abu Sa'id who reigned from 1309 until 1331. Being a student of knowledge himself, he understood the benefits and the attraction and he became dedicated to teaching everything that he knew. His dedication to the art of teaching is shown in his writings, which have evidence that they were intended mainly for al-Banna's students. There are nearly one hundred titles that are cataloged under the name Ibn al-Banna. About half of these books are related to astrology, astronomy, and mathematics, with a focus on algebra and arithmetic. The other half of his textbooks span over a variety of topics, from theology, Quranic studies, law - fiqh - in Islam, logic, surface measurement, rhetoric, talismanic magic, weights and measures, the Islamic division of inheritances, and medicine. He passed away on July 31st of 1321 in Fez, Morocco. Because of his mathematical and astronomical achievements, Ibn al-Banna was posthumously honored when a crater on the moon, Al-Marrakushi, was named after him.

The Works of Ibn al-Banna

Ibn al-Banna was considered to be the last of the Maghribi inventive mathematicians. Inventive, or creative, mathematicians are researchers in the various fields of mathematics who approach problems and give unique answers to them. Ibn al-Banna, himself, was popular in his own time as well as much after his death. His works and life have inspired countless commentaries that were written until the fall of the Ottoman Empire in the early twentieth century.

Within the discipline of astronomy, Ibn al-Banna focused on computing the positions of the planets. He followed the Andalusian tradition, which was related by Zarqali, who was an eleventh century Toledan astronomer. Ibn al-Banna studied these works and proceeded to write a commentary on two separate universal astrolobes and tables regarding planetary motion in an astronomical handbook designed by Zarqali's extensive research on the topic.

The title of Ibn al-Banna's work concerning this research was *Minhaj al-talib fi ta 'dil al kawakib*, which can be roughly translated to *The Student's Method for the Computation of Planetary Positions*. The *Minhaj* also

contains some information from Ibn Ishaq's unfinished works, which had influenced previous astronomical calculations in the thirteenth and fourteenth centuries. This book of Ibn al-Banna's modified previous calculations on planetary longitudes and rapidly became popular within the scholarly community of the time. The tables of the solar equation, planetary calculations, and lunar calculations were not created, but were edited by Ibn al-Banna. Al-Banna added a constant to each equation and table to avoid negative values, which was the first time this technique was ever used in the Maghreb. Although he kept many of the calculations and the tables the same for most of the planets, he changed them entirely for Jupiter and Saturn. In his book, Ibn al-Banna wrote that he had determined that since Jupiter and Saturn have small epicycles, they should be calculated the same way as the moon.

Ibn al-Banna created a summary of the *Minhaj al-talib fi ta 'dil al kawakib* called the *al-Yasara fi taqvim al-kawakib al-sayyara*, which translates to *The Simple Methods for the Computation of Planetary Positions*. In this work, he simplified his tables for the popular astrologers who would be expected to memorize his work during their education. Ibn al-Banna wrote another work summarizing the *Yasara* even further, called the *al-Ishara fi ikhtisar al-Yasara* - meaning *How to Summarize the Yasara*. All three of these works of Ibn al-Banna's - the *al-Minhaj*, the *al-Yasara*, and the *al-Ishara* - were still being used at minimum seven centuries later, at which point no less than three commentaries had already been written about *Minhaj al-talib fi ta 'dil al kawakib*.

Ibn al-Banna was always interested in practical application of his knowledge, more so than conceptual speculation. In his books, he often wrote applications to be used in everyday practice. He wrote about applying geometry to land surveying, solving inheritance partitioning issues with arithmetic and algebra, and using certain procedures for calculating Rumi ciphers used in Maghribi inheritances. These were just a few of the applications that Ibn al-Banna wrote; being a Muslim, much of his work also related to Islam. He wrote about both, the pre-Islamic and Islamic, calendar system, timekeeping in regards to the five daily prayers in Islam, moon sighting for the beginning of new lunar months - specifically for the month of Ramadan in the year 1301 due to a difference of opinion - in the Islamic calendar, and direction for the Muslim prayer to the Kabah in Makkah.

Despite his expanse of knowledge and his literature, there is still much speculation concerning the originality of Ibn al-Banna's various works. Of what has been confirmed as his works, one of the mathematical innovations that Ibn al-Banna is credited for is being the first to consider a fraction to be a ratio of the numerator and the denominator. He also coined the term "weather" in one of his books about meteorological and astronomical data.

Talkhis amal al-hisab, translated to the *Summary of Arithmetical Operations*, and *Raf al-Hijab*, al-Banna's commentary on *Talkhis amal al-hisab*, are considered to be Ibn al-Banna's most famous works. The *Talkhis* is the work that originally introduced some specific mathematical notations. These notations led some historians to the conclusion that algebraic symbolism was first introduced in Islam. Years after Ibn al-Banna's death, Abu'l Hasan ibn Ali al Qalasadi used this work of Ibn al-Banna's to develop the system of using letters in place of numbers.

The *Raf al-Hijab* also included many other mathematician novelties. One such idea was the concept of continued fraction expansion. Continued fraction expansions are used to approximate exact square roots. Below is a depiction of the process that continued fraction expansion takes. It is a continuous, systematic, and repetitive cycle of division:

$$a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{\dots a_3 + \frac{1}{a_n}}}} \quad (1)$$

Carl Friedrich Gauss and Alfred Pringsheim, in separate instances, further simplified Ibn al-Banna's equation. They rewrote the continued fraction expansion with different, but related mathematical notation in an attempt to make it easier to use, with less steps. Ibn al-Banna also found more results relating to the summation of different series. al-Banna focus on series that had differences and exponentials, two of his simplest series are depicted below:

$$1^3 + 3^3 + 5^3 + \dots + (2n - 1)^3$$

and

$$1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2$$

The third of Ibn al-Bannas mathematical innovations listed in *Raf al-Hijab* was that of binomial coefficients with and without relation to statistics. He writes that combinations can be obtained by multiplying the term by its preceding term. The mathematical text below shows the formula written, first in its short form and then in its expanded form:

$${}_p C_k = {}_p C_{k-1} \frac{p - (k - 1)}{k} \tag{2}$$

$${}_p C_k = \frac{p(p - 1)(p - 2)\dots(p - k + 1)}{k!} \tag{3}$$

Binomial coefficients are a small advancement from the Pascal triangle, as it establishes the relation between polygonal numbers and combinations.

Ibn al-Banna's attitude towards astrology is debated between historians who have studied his life. It is agree upon that Ibn al-Banna was interested in astrology in his early life, where he wrote a series of short astrological works that unfortunately lacked originality. These works show that Ibn al-Banna followed an Andalusian-Maghribi tradition and now an Eastern Islamic one. However, his work entitled *Rass 'ala al-Ahkam al-Nujumiyya* - meaning the *Refutation of Astrological Judgements* - written in his later scholarly life shows his beliefs to be closer to the Eastern Islamic tradition. However, it is difficult to affirm that Ibn al-Banna completely lost faith in the authenticity of astrology as the Marinid Sultan Abu Sa'id continued to consult him for astrological matters and his *Minhaj* mentioned previously describes his belief of the mathematical nature of astrology.

Collaboration with other scholars

Of the eighty two texts written by Ibn al-Banna, those that were not about his own mathematics, or about mathematics at all, were still considered to be great in their own context. He wrote an introduction to Euclid's thirteen book series on mathematics called *Elements* - the same book that he once studied as a child, he wrote a book on algebraic text and writing in general, as well as many other works that related to astrology. Though Ibn al-Banna does not claim originality in any of his own works, it is difficult to distinguish how much of the work is actually his own and how much is simply his version of mathematics that was previous dictated and distinguished by earlier Arab mathematicians that has been lost through time. His style of writing often times does lead his audience to assume that his writing is a collection of ideas of other mathematicians in the past.

However, it is believed that the work Ibn al-Banna did on his own, was, in fact, his own - without the influence of his peers.

Historical events that marked Ibn al-Banna's life.

At this particular point in history, the Almohad tribe ruled the area that is now modern day Morocco and southern Spain. In 1212, the Almohad ruler Muhammad al-Nasir was successfully advancing further north in Spain. Initially successful, he was eventually defeated by an alliance formed by the four Christian kings of Castile, Aragon, Navarre, and Portugal in the Sierra Morena. In the Battle of Las Navas de Tolosa, the Christian army's front broke the Almohad lines. However, the individual kings remained too disorganized to profit from their win.

As Muhammad al-Nasir died the next year, he appointed his ten year old son, Yusuf II, to be the next ruler of the Almohad tribe. As with any child king, the lands were ruled by an oligarchy of elder family members, leading nobles, and palace bureaucrats until he was fit to rule the kingdom himself. This oligarchy made sure to remain on good terms with the neighboring Christian kingdoms, in an effort to create a time of peace. For the next fifteen years, the peace agreements the oligarchy had worked to establish remained in place, providing an era of peace for the citizens.

In early 1224, Yusuf II died in an accident without appointing his own heir to the throne. His death began the slow destruction of the once powerful and united Almohad tribe. It began with the palace bureaucrats in Marrakesh. Shortly after the late Yusuf II passed away, they concocted an election led by Uthman ibn Jami that led to the election of Uthman's grand-uncle, Abd al-Wahid I as the new ruler. The brothers of Muhammad al-Nasir were upset with the turn out of the election. Al-Nasir's brothers governed in the cities of Al-Andalus, so they challenged the authority of Abd al-Wahid I. The governor of Murcia, Abdallah al-Adil, then declared himself to be ruler of the Almohad tribe. The brothers helped Abdallah al-Adil in securing the entire land of Al-Andalus, and Abdallah al-Adil's questionable advisor organized the assassination of Abd al-Wahid I and the expulsion of his family.

This coup was the first amongst the Almohads, and the one that began their demise. The Almohads had always kept an image of unity amongst the leaders, always keeping an image of uniformity for their subjects and other lands. The coup shook this facade that had long since been present and opened the gates for other invasions. An army from Castile along with a band of Almohad rebels began besieging cities like Jaen and Andujar in 1225. Coming from the hills, they raided the surrounding regions, and eventually established themselves in Cordova.

Seeing the holes in the Almohad reign, Alfonso IX of Leon and Sancho II of Portugal began their own raids into the Almohad territory. With the Almohad army spread thinly, trying to defend the country's borders, the Portuguese easily reached Seville by the end of 1225. The governors of the city knew they would lose, so they refused to take action. The desperate townspeople were annihilated by the armed Portuguese as they tried to hold onto their land. This event, amongst the many others, lost the people's trust in Abdallah al-Adil and the Almohad leadership.

The collective distrust in Abdallah al-Adil led to uprisings amongst the majority of the people, who began to overthrow the governors of the cities that al-Adil reigned over. In one such instance, the head of one of his governors was sent to the capital, in an effort to show Abdallah al-Adil that the resistance was gaining strength. Though he had a few success stories in this time, he was eventually assassinated by the rebels in October of 1227.

The chaos that followed after al-Adil's assassination led to the spiraling end of the Almohad rule. The

constant shifts in power and disorganization eventually lead to the demise of a once great dynasty. When their own tribe was weak, the Marinid tribes were followers of Cordoba's Almohad caliphs. However, they rose to power as soon as the Almohads were losing theirs. The tribes at the time were ruled by Abu Yahya, who lived in present day eastern Morocco. Initially, the Almohads were able to fight back and push the Marinids into the Rif Mountains, where they resided for nearly thirty years. In 1244 under the command of Abu Yahya, the Marinid tribes were able to conquer Rabat, Sale, Maknes, and Taza - all major defeats for the Almohads. In 1248, the Marinids captured Fez and made it their capital city as well. They captured the rest of Marrakesh from rule under what was left of the diminishing Almohads tribe in 1269, now controlling all of present day Morocco.

Being in control of all of Morocco, the Marinids did their very best to help the surrounding Muslims who were not under the Marinid control. They worked to stop the Christian Spaniards from advancing through the Spanish colony of Granada, in support of the thousands Muslims living there at the time and the Almohads who they had once supported. The confusion as to the birthplace of Ibn al-Banna is a direct result of the friendly nature between the Muslims of Morocco and the Muslims of Granada and their shared desire to restrict the Spaniards.

In 1260, and again in 1267, the ruling Marinid dynasty was attacked by the Spanish Christians trying to invade Morocco. The Spaniards' attempted to invade Morocco multiple times but were continuously unsuccessful in their relentless quest. In fact, the Marinids turned the tide of the numerous attacks completely around and ended up gaining a foothold in modern day Spain, as they became active in the conflict in Iberia between the residing Muslims and Christians. To help the Iberian Muslims, the Marinids successfully gained control of all the trade through the Strait of Gibraltar, where they had occupied Tora, Gibraltar and Tarifa by the year 1294.

The Marinids, who had a major thirst for knowledge and advancement, transformed Fez into their very center for learning and development. They created a university the same university where Ibn al-Banna was educated and taught in his later life - that thrived under the support structure and continuous funding built by the Marinid dynasty. This university was the first of its kind in Fez and in the surrounding region. The Marinids developed the city's residential district, the public buildings, and the principal monuments of the time. The Marinids built their Royal Palace adjoining to the Great Mosque and became entrepreneurs in the success of the sphere of knowledge. Along with all of these advancements in proprietary, they made Fez their administrative and military center. During the time the Marinids were in control of Fez, it became the largest city in the world.

Abu Said Uthman II, the ruler of the Marinids at this particular point in the time period, began construction all throughout Morocco. He built several schools in various locations around the territory in an effort to create a definitive bureaucratic class to undermine the people who surrounded him politically that he did not agree with. His policy was created by the Emirate of Granada, from whom the Marinids had enlarged their army years before.

The very height of the great Marinid rule was under the strong leadership and wise guidance of Abu al-Hasan Ali. Abu al-Hasan 'Ali consistently kept his army very disciplined and very large. They were constantly expanding his borders as well as keeping them safe from outside attackers and invaders. His own bodyguard was filled with at least seven thousand men who rotated serving as his protection at various times throughout his rule. The army had a cavalry that was over forty thousand strong, as well as an abundance of archers that they kept under their wings, keeping their backs safe and covered. Under the reign of Abu al-Hasan Ali, yet another attempt made to reunite the Maghreb was made. It was also under Abu al-Hasan Ali's rule that Ibn al-Banna passed away.

Significant historical events around the world during Ibn al-Banna's life

Emperor Henry, the King of Sicily at the time, called for the Fourth Crusade to begin. He did this in an effort to seize Jerusalem for Christian rule. At this, Pope Innocent III issued a decree for all of the Christian and non-Christian European monarchs to join the struggle to regain the holy land. French barons were particularly touched by this call and proceeded to send their entire armies to aid the Pope, as the French no longer held the credibility that they once did due to continued failures. With the aid of the Venetians, passage across the sea via ships was secured and two hundred boats made their way from the harbors in Europe. Hungary was the first stop for the Crusaders. There, some land was captured for the Venetians as a 'thank you' for their help.

The Christian army's plan had been to first strike the Muslims in Egypt, the center of their civilization and of their society. Yet, the Crusaders never made it to Egypt. Instead, the army arrived at the capital of the Christian Byzantine Empire, Constantinople. They attempted to overthrow the king of the byzantine empire and reinstate another one of their choice, who favored much the same ideals as the crusaders. This stop in Constantinople was only meant to be a quick detour on the way to their destination of Egypt. The reasoning was that a king who was indebted to the Western Christians would be more cooperative and helpful in their quest for the holy land.

The force was able to secure the throne for their choice: Alexius, the son of the former deposed Emperor. Power struggles continued across the Byzantine empire. This prolonged stop ended with the Crusaders eventually losing Constantinople to the regrouped Byzantine government, who were able to put together a force strong enough to beat the Crusaders. The Crusaders never made it to Egypt, and the Muslims remained unaffected by the Fourth Crusade.

Northeast Asia saw the formation of the Mongol Empire under the brilliant military tactician Genghis Khan. Born around 1162, his mother was successfully able to teach him very early in his childhood about the basic survival tactic of forming alliances. Mongolian society at the time was very turbulent and tribal, the only chances for any sort of survival were making alliances with the wealthy and influential tribes in the surrounding region. Genghis's father was the chief of his clan, but was poisoned by his enemy tribes when Genghis was still a young boy. Ostracized, the family struggled to survive in an environment that shunned the widows. A dispute in a hunting expedition led to Genghis killing his half-brother, cementing his place as head of what remained of his family.

Fractious relations led to his kidnapping by a former ally around the age of twenty. After escaping from captivity, he began to build his very own army. More than twenty thousand men rallied under his standard as he set out to destroy the differences between the individual tribes and unite them together under his rule. He was brutal in his approach. Annihilating those who had harmed his family in the past, he ordered older boys and men of those tribes to be massacred. By 1206, central and eastern Mongolia were soundly under his control.

Successful, Genghis Khan continued in his brutal campaign. By then, his eighty thousand man army held no qualms about their continued success. Their brilliant leaders understanding of war led to their repeated victories and significant lack of failures. Khan's understanding of the motivation of the enemy led him to be able to strategize to destroy their motivation, thereby creating an environment for his own success. He was quick to utilize new technologies that were being developed, especially those of the people whom he conquered. He utilized all the resources and advancements of the lands that he now considered his own. Noting his continued triumph, the tribal leaders agreed to follow him and gave him his title of Genghis Khan. Now the spiritual and political leader, he was considered divine and all considered that it was his destiny to rule the world.

The Mongols were soon facing serious food shortages across much of Mongolia. In 1207, Genghis Khan led his men successfully against the Xia Xia Kingdom of China. Following this victory, they set off further towards northern China to vanquish the Jin Dynasty ruling that area. Though their cities were a center of art and scientific advancement, the abundant rice fields were the main attraction for the Mongolians who were becoming deprived of food.

Twenty years of Khan's rule were spent conquering even more people and even more land. Khan spent this time driving his army to expand the limits of his empire in many different directions. Diplomacy was initially utilized to develop a relationship with the Khwarizm Dynasty, whose lands included, but were not limited to, Turkestan, Afghanistan, and Persia. A governor who believed one such diplomatic caravan to be on a spy mission, ordered an attack, ignited the fury of Genghis Khan - which is never a good sign. Shah Muhammad was the king of the Dynasty at the time, and refused to extradite the governor of the area. In fact, he sent the severed head of the Mongolian diplomat who had been killed back as an insult to the Mongols and to Genghis Khan.

Genghis Khan refused to tolerate such annoying and insolent behavior from the people whom he eventually sought to have control over. Calling up his strength in arms, he attacked the Khwarizms city by city, village by village - gaining land like no other. A three pronged attack was personally planned by him, and his soldiers were ordered to be as savage as possible. No city on the warpath was left untouched, effectively causing eminent destruction. Men, women, and children were all killed, even the small domestic animals were not safe from the rage of Khan. The skulls were gathered in great, grotesque pyramids, symbols of the might of the Mongol Empire. By 1221, Shah Muhammad and his heir had been captured and killed, signaling the end of the Khwarizm Dynasty.

Mongol conquests continued throughout much of Asia, marked by the terror they incited in the people they were about to face. When the news came that the Mongols were coming, people panicked and fled - their brutal reputation preceding them wherever they went. Not only were merciless soldiers given quotas of people to kill, but the Bubonic Plague was purposefully spread by the soldiers. These methodologies allowed the Mongols to conquer most of Eurasia at the time. The empire stretched from Korea to Baghdad, and from Russia to the Himalayas.

Genghis Khans glorious efforts allowed the Mongols to eventually capture the entirety of the Silk Road. With so many localities unified under one government, Eurasia saw Pax Mogolica, the Peace of the Mongols. Law and order were very strictly enforced, as well as the consideration for others and for the environment. Any person found to be breaking the rules of the empire was punished, usually by death. It was said that a young woman could walk around safely with a nugget of gold on her head - for theifs feared the punishment and few risked it. Common Mongol law, known as Yassa, was heavily enforced. After so much war and destruction, this peaceful time was a relief for the locals. It allowed for the free spread of ideas and culture throughout the continent.

A very different set of the Crusades continued in Europe, the Crusaders were no longer focused solely on driving the Muslims out of Jerusalem. Their goals had expanded to combat any and all enemies of pure Christendom now. In 1208 to 1229, the Albigensian Crusade took place. In France, there was a sect of Christians who practiced Catharism. Known as the Albigensians, they believed that Christianity should focus on perfection, poverty, and preaching. Their teachings included a dualistic relationship between God (perfection) and Satan (evil), claiming that the physical world was evil and therefore the rule of governments and their laws did not and would never apply to them. Powerful nobles who sought independence from the French king gave protection to the Albigensians. From 1220 to 1229, the military struggle against the Albigensians continued. In 1229, the leading nobles of the revolt signed the Treaty of Meaux-Paris, bringing the region to a state of relative peace.

The stated purpose of this particular crusade of the time was to take control of the pagans in who resided in Transylvania. Yet, the Catholic Church in Rome stood to gain much economic advantage if their efforts succeeded. Denmark, Poland, Sweden, and Christian knights from Germany participated in this victorious endeavor of the Crusaders.

In 1216, Pope Innocent III set into motion the final crusade he would see in his long lifetime. Plans were drawn to yet again invade the Muslims from both land and from sea, similar to the strategy from the Fourth Crusade that failed in Constantinople. Al-Malik Al-Kamil, the nephew of the revered Muslim military hero Salahuddin, successfully defended Jerusalem from the Crusaders and kept the land in Muslim control for even longer.

The year 1229 saw yet another treaty in the mediterranean region. This time it was between Al-Malik Al-Kamil, the Sultan of Egypt at the time, and Emperor Frederick II of the Holy Roman Empire. For the next ten years, the Emperor was granted the city of Jerusalem. Because Frederick had been excommunicated, the land was technically under interdict.

Around the end of the thirteenth century, there were several more attempts to permanently regain Jerusalem. Yet, they were not strong enough to be of much note. The Muslim rulers easily brushed the attackers off. Thibault IV of Champagne and Louis IX of France both crusaded the land, but to no real affect in power. In 1290, a fleet of boats sailed again from Venice. This time, the goal was to defend the last of the Crusader states. The Muslims marched on the capital of the Crusader states, Acre, and secured victory in only seven weeks.

The reputed King John of England came to power after the death of his brother, Richard the Lionheart, in the Crusades that spread across Europe. As king, he used his power to extort large sums of money from his lords and barons - who did not appreciate such extortion. He was at war with France, and needed the funds to keep the war going, but only angered those whose respect and loyalty he needed to remain a unified and strong nation.

Yet, these policies only served to incite the legendary barony, who were once under his leadership and control, severly against him and his crown. In 1215, seeking to make peace with the rebels, King John issued the Magna Carta which is latin for the Great Charter. The Magna Carta marked the first of all time in English history where the king was also subject to the laws he subjugated his subjects to. It both addressed the many grievances the barons held against the King, but also held components that would become the foundation of future democratic charters internationally. These include the 1791 Bill of Rights in the United States of America, the 1948 Universal Declaration of Human Rights, and the 1950 European Convention on Human Rights.

The Magna Cartas promises to the barons included the protection of their church rights, protection from illegal imprisonment, swift justice, and limitations on taxes to the king. All of these protections were issues the barons had faced of the years. After repeated repeals and reissuements, the Magna Carta became part of Englands statute law.

The then pope of the Roman Catholic Church George IX became concerned at the increasing heresy he was seeing throughout the land. In response, he established the Inquisition. The Inquisition was a council within the Roman Catholic Church. Its main goal was to investigate the multitude of claims of heresy spread across Europe.

After the Albigensian Crusade, the Pope was ready to take real action. He established a papal Inquisition and sent friars for that purpose to Southern France. Once the inquisitors arrived, one month was given for the guilty to confess their sins. Upon doing so, they were forgiven with little punishment. However, for those who chose to not make penance, an almost certain guilty verdict awaited. They were not given the name of

the individual who accused them. Instead, they were given an opportunity to name their enemies. This would nullify testimony from any of these people. The accused were brought to secret trial before a bishop and some laymen. It was common for them and their witnesses to be tortured. Appeal to the pope was their only means of recourse after a guilty verdict came down.

Being declared guilty meant being handed over to local authorities. Heresy came to be used as a weapon for heads of state. They would declare their enemies heretics and enjoy the consequences. This was because in the case of civil leaders, territory was confiscated and handed over to the Church. This also led to many cases of graft and blackmail. It added a layer of suspicion to the Inquisition.

Renaissance, rebirth, occurred directly after the Dark Ages of Europe. The common people were done. They were done with death, disease, and misery. They were done with the hard lives they led. They wanted something new. They wanted culture. Liveliness and excitement were in high demand. Ideas were high in supply. The Renaissance originated in Italy before travelling north to the rest of Europe.

Here was a time where the Ancients were brought back to life. Classical scholarship became popular. Discovery was the fuel of everyday talk. There were whole continents waiting to be explored. Astronomy was reenergized using the Ptolemaic system. The feudal system was in great decline. There was a surge in commerce and invention. Paper, printing, the mariners compass, and gunpowder were all brought to Europe. The stagnation of the Middle Ages was exchanged for Classical methods and wisdom. Key to the ideas of the Renaissance was humanism. Characterized by a commitment to human nature, unity and compatibility of worldly truths and theological ideas (syncretism), the dignity of man, and a rebirth of the lost human spirit and wisdom, humanism became the driving theme of the Renaissance. For a Europe emerging from the Dark Ages, this was a much needed cultural energizer.

Significant mathematical progress during the Ibn al-Banna's lifetime

Because of stunted communication, thousands of miles away, Qin Jiushao was making mathematical history that Ibn al-Banna was not exposed to. Qin Jiushao also studied astronomy and mathematics. He resided and studied in the then capital of China, Linan, which is modern day Hangzhou. In 1247, Jiushao wrote his only book, *Shushu jiuzhang*, roughly translated to *Mathematical Writings in Nine Sections*.

The book is combination of nine different problems relating to construction, military affairs, surveying of remote objects, computations of calendars, fortifications, proportions, interest calculations, surveying of fields, meteorology, commercial affairs, area and volume calculations of plane and solid figures, and indeterminate analysis. Every question that he encounters in his book is followed by three things: a general solution, a description of the calculations, and a numerical answer. He is most credited for his work on finding the algorithm for calculating higher degree polynomial equations, using successive approximations, and for finding the solution to simultaneous linear congruencies, using different modules. Qin Jiushaos methods of successive approximation were rediscovered in 1802 in Europe and named the Ruffini-Horner method even though Jiushaos record is the earliest description of this algorithm.

Also in China during this same time period was Zhu Shijie, also commonly known as Chu Shih-Cheih. He is known amongst the Chinese for having united the different mathematical traditions of northern and southern China. He is thought to have been a native from the present day Beijing region. However, his thirst for adventure and experiences lead him to travel the country as a student of knowledge and an itinerant teacher. Much of Shih-Cheih's fame is surrounded around two of his publications.

The first was published in 1299, called *Suanxue qimeng*, meaning an *Introduction to Mathematical Science*. As it may seem, this book was an introductory mathematics textbook. It discussed many different concepts

from elementary arithmetic all the way until algebraic calculations. The text book is written in a manner that encourages memorization, with poetic verses to facilitate the learning. This book follows the style of southern Chinese mathematics.

Chu Shih-Cheih's second important publication was published in 1303, titled *Siyuan yujian*, meaning *Precious Mirror of Four Elements*. Now focusing on northern Chinese mathematics, he completes the generalization of a certain algebraic computation that is completed with counting rods. This method began with Li Ye, another famous mathematician of the twelfth century. This method was developed to calculate dimensions of simple geometric shapes, give supplementary data such as the shapes area or volume. Shih-Cheih demonstrated that this method can be used to solve a wide variety of questions, including, but not limited to, polynomials and other equations. *Precious Mirror of Four Elements* also contains a diagram known as Blaise Pascals triangle that inspired Shih-Cheih's discovery of a combinatoric identity.

Connections between history and the development of mathematics

The crusades really highlighted the influence of the Catholic Church on medieval life. This influence was strong enough to call men to leave their wives and children and journey to war. There was no guarantee that they would return home. A body with such strong influence on its subjects was strong enough to decree what should be studied and what should not. Anything that disagreed with the Bible or the Church's teachings was immediately outlawed. It was called heresy, and its propagators were subjected to punishment. A famous example of this is Galileo.

Naturally, this greatly dissuaded great minds in Europe from exploring and inventing. There was no reason to risk their lives for some advancement that may or may not take hold. This relates to Maslow's Hierarchy of Needs. Safety and belonging are a prerequisite for enlightenment, the section where discovery and invention fall.

As cruel and terrible as Mongol rule was, it was very effective in forming a cohesive and safe society. So long as one obeyed the laws and did not offend the magistrates, there was safety. This allowed the many discoveries of the Muslim nations to spread across the land. Mathematical advancements achieved by the Muslims were spread to Africa, East Asia, and Europe. With safety is present, ideas travelled quickly.

The Renaissance was the European equivalent of the Islamic Golden Age. There was untold advancement. Inventors finally felt free to let their ideas loose and their thoughts run wild. No longer did they have to worry about stepping out of line with the Church. The Church was losing its influence with the Reformation. Free at last, new ideas and inventions were regular occurrences.

Ibn al-Banna was in the heart one of the greatest periods of mathematical innovation and a time that was prime for success. Fez was the largest city at the time and the advancements in mathematics that occurred in the time of Ibn al-Banna are incredulous. The university fostered by the Marinid tribe created an environment for success; and environment that led to over eighty books being written by one man alone.

Remarks

Ibn al-Banna was a brilliant mathematician who discovered worked on math that modern mathematics could not have progressed without. His understanding of square roots, series expansion, and what is today known as elementary statistics was unheard of at the time. His understanding and determination for the art needs to be commended, for it is the foundation that built the matter for generations to come.

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