Gerolamo Cardano

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Gerolamo Cardano's Life

The 16th Century Italian Renaissance produced many great mathematical minds, all of whom in one aspect or another contributed greatly to the many different fields of study in mathematics. Among the most notable and most influential figures of this time was Gerolamo Cardano, an Italian mathematician, physician, and philosopher, who made great strides in the fields of Algebra and Statistics. He is widely considered by many to be centrally important to the discovery and publication of the general solutions to cubic and quartic equations, as well as being credited as the founder of classical probability theory. He also mastered several ingenious, creative feats of engineering, which had applications in many great inventions, both of – and immediately following – his time. It is because of his great contributions to these fields that he is considered to be an important historical figure that was very influential to many of the great mathematicians and scientists succeeding him, including such giants as Blaise Pascal, Sir Isaac Newton, Pierre de Fermat, Gottfried Leibnitz, Joseph-Louie Lagrange, and Carl Friedrich Gauss. Besides Cardanos triumphs, his life was also marked by terrible tragedy. Pitted from birth against the odds of achieving financial or educational success, for reasons both external and personal, he was never without challenge or strife. However, handicap can often be one of the best motivators. So, it is perhaps because of this strife, arguably, that Cardano was able to rise from the depths of despair and produce his best work.

Gerolamo Cardano was born on the 24th of September, 1501 in Pavia, Italy, the height of the Italian Renaissance, as the illegitimate child of the lawyer and mathematician Fazio Cardano and his mother Chiara Micheri. While Fazio made no notable mathematical contributions to his credit, he had lectured at the University of Pavia on the subject of geometry for some time, while simultaneously practicing law in his village. By the time Fazio had reached his fifties, he had met the lovely but desperate Chiara Micheri – a young widow in her thirties with three children, all struggling to eat and in great need of financial support. Fazio, with no intention to commit to Chiara, had a brief tryst and ultimately impregnated her only a short time after their meeting. At this same time, coincidentally, the Plague had hit Milan, Italy, and Chiara was persuaded to escape from the town, leaving her children behind, to stay with some of Fazios wealthy friends in Pavia. Shortly after moving to Pavia, Chiara received the news that her three children had all died from the Black Death Plague in Milan. It was around this time that Gerolamo was born, into a world of uncertainty that would set the stage for not only his lifes greatest achievements, but also his greatest downfalls.

At a young age, Cardano began learning his destined trade by becoming his fathers law assistant for a brief period. But despite his youth, he was weak, mainly from malnourishment due to poverty, and was somewhat ineffective for many tasks. Soon, Fazio needed to enlist the help of two of his nephews, forever diminishing the young Cardanos taste for the law and for simpler work. However, in his free time, Fazio gave Cardano a basic education in mathematics, which left the young student yearning to pursue an academic career in the field. However, when it came time for Cardano to choose his formal field of study, he refused to follow in his fathers footsteps as a lawyer and a lengthy argument ensued. Inevitably, Cardano was allowed to enroll at the University of Pavia in the year 1520 to study medicine. Once Cardano had grown a bit older, tragically, war broke out between France and Spain, and he was forced to move to the University of Padua. Shortly after this transition, Fazio Cardano died of old age, which led to the young Cardano – now widely known as a brilliant and diligent student – to make a bid for the rector of the University. However, because of his confrontational and outspoken nature, Cardano was not well-liked by his peers, which proved to be a challenge for him in the election. Yet despite many roadblocks, Gerolamo won the coveted title of rector, beating his primary opponent by only one vote. He completed his formal education in medicine at the University of Padua in 1525, receiving his medical doctorate at the age of 24.

Cardano struggled initially as a physician in terms of credibility in Milan. In 1525, prior to completing his

degree, he had applied many times over to the College of Physicians in Milan, but was repeatedly rejected for his illegitimate birth and his reckless and confrontational reputation. Thus, at the recommendation of a friend, he set up his first medical practice in the small town of Sacco, very near to Padua. However, his practice was overall quite unsuccessful due to a lack of patients, even though he continued with it for many years, doing what he could to provide for himself. In the latter half of 1531, Cardano married Lucia Bandarini, the daughter of his neighbor, and had two sons and one daughter with her. However, this complicated things for him even further, since he now had to support both his family and himself with what little he made from his dwindling practice in Sacco. In 1532, the couple and their children moved to Gallarate, a town very near to Milan, in another attempt for Cardano to join the College of Physicians, but to no avail. In 1533, Cardano reduced himself to gambling in order to pay his bills; but even this went so badly for him that he was forced to pawn Lucias expensive jewelry for whatever money he could get for it. In one last desperate attempt to turn things around, Cardano and Lucia and the children moved to Milan. It was there, unfortunately, that they had even worse luck than before, and the family sank further into poverty. Fortunately, his father had been a well-known man of Milan, and in a stroke of luck, Cardano was given a position at the Piatti Foundation as a lecturer on mathematics – ironically the same position once held by his father Fazio.

In 1939, while holding this position, Cardano wrote some of his first mathematical works, the most notable among them being Ars Magna (lit. Great Art). During the composition of this work, he and his student Ludovico Ferrari had heard that a contemporary mathematician in Milan, Nicolo Tartaglia, had uncovered a general rule for solving cubic equations. Cardano was determined to include this rule in his book, and after many attempts to contact and extract this information from Tartaglia, he had still failed to learn the secret rule. One evening, however, Cardano had invited Tartaglia over for dinner, and after a bit of flattery and pandering, he had convinced him to give away his secret in the form of a poem. This titillating agreement was based on the condition that Cardano would only publish the secret after Tartaglia himself had publicly exposed the finding. Cardano initially had the full intention to uphold this advantageous agreement, but he ultimately did not. After discovering that another mathematician, Scipiorre del Ferro, had solved the problem, Cardano set out to learn this new method. Once learning the secret to solving cubic equations, as taught to him by Ferros close student, Antonio Fior, Cardano felt that he no longer had an obligation to uphold his agreement with Tartaglia, and subsequently published both solutions in Ars Magna.

During this period, Cardano engaged in heavy and rigorous mathematical research, unparalleled to any mathematical work he had done while working as a physician. He became the first mathematician to formally use numbers less than zero in his work, and spent almost six years of study on finding radical solutions to both quartic and cubic equations. Interestingly, one problem Cardano had with the formula for solving cubic equations was that it would sometimes yield results that involved the square root of negative numbers. This, of course, meant that he was dealing with imaginary numbers. However, he, as well as his contemporaries, did not necessarily understand any of the properties of such numbers; they were only coincidentally discovered through his main work on polynomial functions. As Cardano continued to excel in the field, Tartaglia had heavily begun regretting sharing his secrets with Cardano.

With a gambling addiction and a propensity for using his math skills to play the odds, in 1540, Cardano resigned from his post as lecturer at the Piatti Foundation, and spent the next two years of his life gambling obsessively and playing chess. In 1545, Ars Magna was finally published, and it soon became known as not only one of Cardanos greatest works, but also one of the greatest mathematical works of the time. Cardanos reputation for gambling and his great mathematical ability made him both famous and infamous as both reputations swirled during this time. Cardanos career reached, arguably, a bittersweet boiling point in 1546, when he faced both the death of his wife Lucia and his long-awaited admission to the College of Physicians in Milan. This admission apparently took precedence over the despair he felt about death of his wife, as he spent the following decade doing incredible work in medicine. Most notably, he cured the Archbishop of St. Andrews in Scotland of his crippling asthma, a task for which he received immense fame and a large fortune.

Upon his return, Cardano was almost immediately offered a position of professor of medicine at the University of Pavia, and gained many wealthy patients who bolstered Cardanos own personal wealth and security.

It was after this period that Cardanos life went into an irreparable state of disarray. In 1557, Cardanos son Giambatista married Brandonia di Seroni. This marriage would later prove to be troublesome, as the di Seroni family only sought to get what money they could out of the Cardano family, all while Brandonia publicly embarrassed Giambatista by claiming that his sons were illegitimate. This eventually drove Giambatista to madness, poisoning his wife and confessing the crime soon after his arrest. Despite all the best lawyers Cardano could afford, his son was executed on the 13th of April, 1560, after a failure to settle with the di Seroni family. Cardano was now hated by the public, being the father of a convicted murderer. This caused him to flee to Bologna, where he took up a position as a lecturer of medicine at the University of Bologna. It was during this time, around the year 1563, when it is believed that Cardano had written the first formal mathematical work on the theory of chance and probability, titled Liber de Ludo Aleae (lit. Book on Games of Chance), though the work wasnt published until long after his death in 1663. Shortly after assimilating into Bologna, Cardano quickly made many enemies, and became hated for his confrontational personality style. The final greatest sadness of his life, according to Cardano, was his son Aldos descent into gambling addiction and crime. In 1569, Aldo had reached such a low that he resorted to stealing a large sum of money and also a large amount of jewelry from Cardanos house. This led to, regrettably, as Cardano himself states, him turning in his youngest son Aldo over to the authorities.

Towards the end of his life, in 1570, Cardano wrote a highly controversial Horoscope for Jesus Christ, as well as a book praising the Roman Emperor Nero, who tortured many Christian martyrs during his reign. For this, Cardano was imprisoned by the Roman Inquisition. However, he was only kept for a few months, because the consensus of the people was that he was treated too harshly for a man who otherwise completely supported the Catholic Church. After being released, Cardano travelled to Rome, and, surprisingly, was forgiven by the Pope and offered membership at the College of Physicians there, where he remained for the rest of his life. In line with his gamblers mystique, it is said that he correctly predicted the date of his own death to the very day, the 21st of September, 1576. This controversy is debated, as he may have committed suicide in order for this prophecy to have been true in retrospect.

Cardano's Mathematical Works

Gerolamo Cardano was certainly like many other mathematicians of his time, in the sense that he used work which preceded his own to study the properties of numbers and relationships. He was unlike the other mathematicians of his time, however, in the sense that he viewed counterintuitive mathematical concepts equally worthy of study as potentially more practical – or more useful – mathematical concepts. The central concept that he laid the fundamental framework for was the Theory of Probability. It wasnt until long after his death that the classical theory of probability was formulated based on his work, by Blaise Pascal and Pierre de Fermat. He is also noted as being the first mathematician to use negative numbers systematically in a work, as well as one of the first to recognize the existence of imaginary numbers. Before Cardano, no other mathematician had taken the initiative to formally and systematically study the concept of probability perhaps fueled by his love of gambling, and playing the odds. The concept of chance, up until the publication of Liber de Ludo Aleae, was largely just a symbolic or unquantifiable concept. With the production of his work, he highlighted the importance of identifying a sample space with equally likely outcomes. He also was the first mathematician to define odds as the ratio of favorable to unfavorable outcomes in a game of chance, which implies that that the total probability of an outcome is the ratio of favorable outcomes to all outcomes in the sample space. In modern notation, Cardanos definition of odds can be expressed as:

$$o(x) = ((p(x)) = (q(x)))$$

Where p(x) would be the integer number of favorable outcomes of event x and q(x) would be the integer number of unfavorable outcomes of event x. We would say, by this definition, that the odds are in ones favor if:

$$o(x) > 1$$

And the odds are out of ones favor if:

$$o(x) < 1$$

From this we see that the definition of total probability of event x occurring is not far off from Cardanos notion of odds in a game of chance.

Collaboration With Other Scholars

Cardano is also responsible for the formalization and publication of the general rule for solving a cubic function of the special case:

$$x^3 + px = q$$

to which, through the analysis of the work of both his student Ferrari and his contemporary Tartaglia, he was able to provide the solution, which is of the form:

$$x = (1/2q + ((1/2q)^2 + (1/3p)^3)) + (1/2q - ((1/2q)^2 + (1/3p)^3))$$

When we look at this solution, we can see where Cardano would have been puzzled over the occurrence of imaginary numbers in certain solutions. If we take the discriminant of the solution:

$$(1/2q)^2 + (1/3p)^3$$

And make it so that:

$$p < 1$$

Then we have that taking the square root of the discriminant may result in the square root of a negative number.

This disturbed Cardano, because certainly, he thought, there must be practical solutions that can be calculated out of this, but the concept of the square root of a negative seemed impossible to work around. Famously, out of anger, Cardanos rival Tartaglia, tried to misdirect Cardano, by claiming that the occurrence of these negative roots is evidence of an erroneous theory, and that surely something must have been wrong. Of course, neither Cardano nor Tartaglia had any understanding of imaginary numbers throughout their careers; however, this occurrence is important to note, because it depicts how imaginary numbers arise naturally out of the study of algebra.

Local Historical Events That Marked Cardano's life.

It is important to discuss the historical events which took place in the same geographical area and time period that we would find Cardano in while his mathematical research was taking place. There are many circumstances throughout Cardanos life that were purely external in nature which led to Cardanos life and research changing drastically. The first and most pressing condition of Cardanos life is that he was both born and raised in the middle of an outbreak of the Bubonic Plague, more commonly referred to as the Black Death, in Renaissance Italy. The Black Death in Europe was responsible for the death of 6

The second most pressing external event which affected Cardanos life is the Italian War of 1521-26, as it led to the closing-down of the first university that Cardano attended, the University of Pavia. This university was forced to close as a safety precaution. The French and Spanish empires at the time used much of Italy as the battleground for their conflict, and as a result, many institutions had to temporarily close their doors to avoid any unnecessary damage to Italys infrastructure. Cardano was, as a result, forced to relocate to the University of Padua, the institution of which he eventually became the rector. Had Cardano not become the rector at this time period, he would have been left in a very unfortunate economic state early on, seeing that his father had died at around the same time. Since Gerolamo was an illegitimate child, he was never named as an heir to the Cardano fortune, so had he not become the rector, his education could have very well been cut short, and he would not have been able to advance without the proper funds. An uneducated man of his time almost never would have been able to make any publications or contributions to academia, so had Cardano not been able to attend the University of Padua, he might have never come to produce the mathematical works that awaited him later in his career.

The third most pressing event, the Roman Inquisition of the latter half of the 16th Century, affected Cardano towards the end of his life, as an elderly man. Cardano was fully supportive of the Roman Catholic Church, however, he produced in his twilight years a few works which the inquisition deemed heretical. The first of these works was a horoscope casted for Jesus Christ, which was considered by many to be blasphemy. The second of these works was a book that glorified the Roman Emperor Nero, an emperor most notable for the persecution and torture of Christians during his reign. Because of these works, the Inquisition imprisoned and interrogated Cardano over the span of several months. Their treatment of him was so cruel, that the public pressured the inquisition to release him earlier than most would have. Upon his release, he traveled to Rome, and the Pope was so apologetic that he was offered an immediate membership of the College of Physicians, as well as a pension. Had he not had such reparations, he might not have been in the proper standing to publish his latest works, including Liber de Ludo Aleae, the work that established, formally, the first mathematical treatment of probability.

Significant Global Historical Events During Cardano's life

What is most important to note about the world during Cardanos time is the fact that he was born during the peak of the European Renaissance, a continent-wide uplifting of European art, music, culture, and science that succeeded the European Dark Ages, a period of time where almost no progress took place in that part of the world. The Renaissance was the perfect time for one to pursue mathematics, because the theories and concepts of algebra were, because of the Arab scholars of the time, just coming to Europe from the East, and many of its properties were either unclear to many Europeans, otherwise unstudied, or completely unknown. This was advantageous to eager young mathematicians of the Renaissance, because, for the first time as well, a type of international mathematical formalism began to arise as trade and the communication of ideas amongst advanced nations of the time began to increase exponentially.

From an even broader historical perspective, we see that the 16th Century was a time where Western culture as a whole had begun to reject the idea that the works of the ancients were pillars of fact. Before this pivotal time, known as the Scientific Revolution, Aristotelian views of the universe were very popular; this impeded hugely on scientific and mathematical progress, because the works of Aristotle and similar Greek philosophers were highly subjective, and in no work of the ancients do we see a formal, simple, or rigorous system describing how methodical research and inquiry ought to be done. Only once one reaches the 15th and 16th Centuries do we see the dawn of the modern scientific process with scientists and mathematicians such as Galileo Galilei, Nicholas Copernicus, Sir Isaac Newton, and Francis Bacon establishing systems and philosophies all centered around the idea of studying nature through logic and reason, as opposed to superstition or intuition. This extends, of course, to the fields of mathematics, because for the first time, the mathematical community had to invent many formalisms and standards in order to more easily and more effectively collaborate with mathematicians of other nations and cultures.

Taking into account that Cardano was born during both the European Renaissance and the Scientific Revolution, we can see how his work in the course of his life was a natural product of the two. It took both the Arab mathematicians bringing over the knowledge of algebra during the Renaissance, and the ability to formally and systematically implement reason and logic into scientific studies which was brought by the Scientific Revolution, to truly drive the study of mathematics forward in the West and, arguably, to accelerate it into the boom in mathematical discoveries that took place in the succeeding centuries, with such notable mathematicians as Gauss, Fermat, Pascal, Euler, and Laplace deriving inspiration for their work from this period of time. Cardano was, in one aspect, a product of these two events, and though he was very much still a man of his time, with limited knowledge, he was a part of the generation of mathematicians who laid the essential framework for all of modern mathematics.

Significant Mathematical Progress During Cardano's Lifetime

Having been born and raised in the early 16th Century, there was certainly an immense amount of important mathematical progress being made during Cardanos time. Specifically, while Cardano was at his peak in terms of mathematical research during the 1540s, much progress in mathematics as a whole was being made across the Western world, most of which can be attributed, in one way or another, to the Arab and Islamic mathematicians of the East.

One important change that took place during Cardanos time was the formal switch from roman numerals to the Arabian decimal system. Though many groups were resistant at first to switching the systems of counting, most over time saw the merit and the usefulness behind it, as you didnt need an arbitrarily large number of individual symbols to represent one number, and basic mathematical operations, such as addition and subtraction, were easier to perform.

While algebra had already come over in part to Europe from the East, it was still largely without a formal notation. In order to express a simple equation, such as y = mx + b, one would have to speak an entire grammatical sentence, complete with syntax, to express the idea that there is an unknown that needs to be solved for. During the 15th and 16th Centuries, however, because of the spread of algebra throughout the world and the excessive need for mathematical formalism, many mathematicians began to abbreviate the unknowns in terms of variables, something that the Arabian mathematicians of the East didnt make the effort to do, so long as their algebraic process was contained within the confines of Arabic. As soon as this was no longer the case, a more abbreviated and symbolic system was required in order to destroy cultural roadblocks when communicating mathematical research to a mathematician of a different nation or culture.

Overall, we see that, interestingly, while algebra could be researched in Europe at the dawn of the 16th Century, it could also be improved upon; and its through this very idea that we saw the rise of mathematical formalism and the swearing-off of language-specific mathematics, which weve seen in every era up until this point. And its because of this new basic mathematical formalism arising that it became easier for mathematicians of the time to do more abstract and rigorous research in the relationships of knowns and unknowns.

Connections Between History and the Development of Mathematics

The relevant mathematical progress made by Cardano can be dichotomized into two fields: Algebra and Probability and Statistics. In terms of algebra, Cardano undisputedly advanced the field with his work on cubic and quartic equations. And while Cardano made little tangible progress in probability theory, he was responsible for laying the framework of the subject, which would subsequently be picked up by mathematicians who succeeded him, and be turned into a full, formalized field of mathematics.

The amalgamated works that Cardano collected on Binomial Theory and solutions to cubic and quartic equations, along with his own research, were integral to the field of algebra. Simply publishing and displaying the solutions to such equations gave much insight, alone, as to how polynomial equations are generally solved. But Cardanos study of the cubic and quartic functions also brought many interesting points of discussion that could be extrapolated from these solutions, such as the existence of imaginary numbers, or the practical applications of numbers with a value less than zero. Both of these concepts would, in the centuries immediately following, be studied rigorously, leading to a wider range of proofs for a wider range of problems.

Cardanos framework for the mathematical treatment of chance and probability marks the starting point, essentially, for all of inferential statistics. Though many mathematicians who preceded Cardano have discussed the idea of chance, he was the first to discuss the idea empirically, and provide a systematic, mathematical treatment for it. Its through this distinction that we identify Gerolamo Cardano as the father of probability theory, and the reason why his work on games of chance and the study of odds is considered highly influential in the history and progress of all of mathematics.

The reason why Cardano was able to accomplish all of these things is, arguably, because this was the first time in history when mathematics was beginning to shed cultural barriers and become a truly socially neutral human task that transcends notions of language and other convoluted information. Rigorous algebraic progress requires ones mental workspace to be free of clutter, and this is essentially what the mathematicians of this period sought to achieve through the formalization and subsequent rigorous study and dissection of the various components of algebra. Though not all components of modern mathematical notation were there, there was still a significant move to the symbolic, which made all mathematicians research of the time much more rigorous, and many times more effective.

Remarks

I feel very fortunate that I was able to study a Renaissance mathematician; this was a time of such progress in all fields, so to see, through the eyes of a mathematician, the framework for many important fields of mathematics laid down in simple terms is nothing short of amazing. I think it really shows how truly beautiful mathematics can be when you view it as a subject that transcends creed and culture. Mathematics is a very universal subject in my opinion, and after writing this paper, Im inclined to agree even more.

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