Niels Henrik Abel was born on August 5, 1802, on the island of Finny—which is near Stavanger, Norway. Abel was born to Soren Georg Abel and Anne Marie Simonsen. He grew up in Norway, and was entered into the cathedral school in Oslo at the age of thirteen. Apparently—like many geniuses both before and after his time—he was thought to be a subpar student for almost his first two years at the school. However, his life changed dramatically when a new mathematics teacher, Bernt Michael Holmboe, arrived at the school. Holmboe recognized Abel's intellectual aptitude quickly, and exposed Niels to complex mathematics. Under Holmboe's tutelage Abel studied the works of Newton, Euler, Gauss, and Lagrange. Niels' promising future was almost jeopardized when his father died in 1820, as poverty would have prevented him from attending the University of Christiania. Holmboe managed to allow Abel to avoid this problem by helping win a scholarship to the university. Abel entered the university in 1821, and by 1822 he had already earned a preliminary degree. By the time of his graduation he was Norway's foremost mathematician. While staying in Copenhagen in 1823, Abel met Christine Kemp. In 1824, Christine moved to Norway with Niels and the two were engaged.

Abel spent his whole life in poverty. His parents were both drunks, and the family never had any money when he was a child. His mathematical successes did little to improve his financial situation. His financial troubles also frequently managed to interfere with his work. One famous example of this involved his proof of the impossibility of finding a general solution to the quintic equation. He self-published the proof, and was forced to limit the proof to six pages to reduce the costs of printing. While the proof remained valid, the published proof was far more difficult to understand than Niels' extended version of the proof. Niels ability to travel was also frequently limited unless he was able to receive grants or scholarships. He came tantalizingly close to having this problem solved for him when he was 27. His friend August Crelle had secured appointment for him as the professor of mathematics at the University of Berlin. Unfortunately, Abel never received this good news, as he fell ill on a ski trip with his fiancee and died on April 6, 1829.
Abel’s mathematical works

One of Abel’s first mathematical accomplishments came at the age of sixteen, when he produced a proof of the binomial theorem—which states that the binomial coefficients are none other than the combinatorial numbers, $\binom{n}{k}$, and the coefficient of the first term is always one, and the coefficient of the second term is the same as the exponent $(a+b)$. While Euler had previously proved that binomial theorem held true for rational numbers, Abel’s proof extended binomial theorem to all numbers.

The theorem states that the binomial coefficients are none other than the combinatorial numbers, $nCk$. The coefficient of the first term is always $1$, and the coefficient of the second term is the same as the exponent of $(a + b)$

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

One of Abel’s most significant mathematical accomplishments was his proof that there is no general solution in radicals to quintic equations—or any equation polynomial equation of greater than fourth degree—with arbitrary coefficients. This was an extremely important proof, as mathematicians had spent almost four thousand years pursuing the general solutions to polynomial equations of various degrees. For first degree polynomials, in the form $ax+b=0$, the solution is almost trivial: $x=-b/a$. However, for second degree polynomials(in the form $ax^2+by+c=0$), the general solution—known today as the quadratic formula—is more complicated.

The Babylonians are credited with discovering the quadratic formula in about 2000 BC. The cubic formula is much more complicated than the quadratic formula, and was discovered about 3500 years later. Girolamo Cardano is credited with discovering the general solution of third degree polynomial equations(in the form $ax^3+by^2+cz+d=0$) in 1545. The general solution for quartic equations is the most complicated of such solutions. Rather ironically, it was actually discovered about five years before the general solution of cubic equations. However, it could not be published before the cubic formula because it relied on the solution of the cubic formula. Lodovico Ferrari is credited with discovering the quartic equation, and he and Cardano published their solutions together in 1545. Abel’s proof in 1824 effectively ended a problem that mathematicians had faced for almost four millennia. The process by which he discovered his proof is almost definitely more important than the proof itself, as it led him to invent group theory—which has led to advancements in mathematics and physics. Ironically, he originally believed that he had found the general solution of the quintic equation, and it was only after he found the mistake in his original proof that he set about proving the impossibility of finding the general solution.
Abel was the first person to solve an integral equation—or an equation in which the quantity to be solved for lies within an integral sign. He first employed an integral equation when trying to solve a problem today known as the Abel Problem in 1823. The problem required one to find in a vertical plane \((s, tao)\) a curve such that a point moving along it under gravity from rest, starting from ordinate \(x\), will meet the \(tao\)-axis after a time \(T=f(x)\), where \(f(x)\) is a given. The solution of this problem yielded the first integral equation, shown below.

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\[
\begin{align*}
x_1 &= \frac{b}{3a} \sqrt[3]{\frac{1}{2} \left[ 2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(2b^3 - 9abc + 27a^2d)^3} \right]} \\
x_2 &= \frac{b}{3a} \sqrt[3]{\frac{1}{2} \left[ 2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(2b^3 - 9abc + 27a^2d)^3} \right]} \\
x_3 &= \frac{b}{3a} \sqrt[3]{\frac{1}{2} \left[ 2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(2b^3 - 9abc + 27a^2d)^3} \right]}
\end{align*}
\]

Abel introduced Abel’s summation formula—which is used to compute series. His formula is extremely useful for mathematicians who specialize in number theory, and it influenced the work of many famous mathematicians. The Euler-Mascheroni constant, the representation and reciprocal of Riemann’s zeta function all were developed with the use of Abel’s summation formula.

\[
\begin{align*}
\int_{x}^{\infty} \frac{\phi(s)}{(x-s)^{\alpha}} \, ds = f(x), \quad \alpha \leq x \leq b,
\end{align*}
\]

Shown below is the generalized Abel integral equation.

\[
\begin{align*}
\int_{0}^{x} \frac{\phi(s)}{\sqrt{x-s}} \, ds = f(x),
\end{align*}
\]

In 1826 Abel introduced Abel’s summation formula—which is used to compute series. His formula is extremely useful for mathematicians who specialize in number theory, and it influenced the work of many famous mathematicians. The Euler-Mascheroni constant, the representation and reciprocal of Riemann’s zeta function all were developed with the use of Abel’s summation formula.
Abel also worked extensively with groups—sets whose elements satisfy the associative property for multiplication and addition while satisfying the identity property as well. Groups that satisfy the commutative property for multiplication and addition are referred to as abelian groups in his honor.

\[
\sum_{1 \leq n \leq x} a_n \phi(n) = A(x)\phi(x) - \int_{1}^{x} A(u)\phi'(u) \, du
\]

Abel’s Collaborations

Abel’s first collaborations came with his teacher, Bernt Michael Holomboe. Holomboe was the first person to recognize Abel’s immense mathematical aptitude, and was vital in ensuring that it would not go to waste. Holomboe introduced Abel to advanced mathematics, and without Holomboe, Abel would never have been able to afford studying at the university. However, it should be mentioned that eventually Holmboe’s presence as the professor of mathematics at the university would hinder Abel’s attempt to be appointed as a professor. After Holomboe, Abel’s most important collaborations were with August Crelle. Crelle and Abel met in Berlin in 1825, and quickly developed a friendship. Their friendship was timed well: Crelle commenced publishing a mathematical journal—one of the first journals to focus solely on mathematical research—not long after meeting Abel. Much of Abel’s work would be published by Crelle. Crelle frequently attempted to aid Abel financially: he offered him editorship of his journal, helped Abel search for scholarships and loans, and even managed to get him a professorship—albeit too late.

Abel did not have many positive interactions with other mathematicians. He sent his impossibility proof to Gauss, but Gauss was dismissive of it and refused to open it. However, he did not lose his admiration for Gauss, and he was quoted as saying of him: ”He is like the fox, who effaces his tracks in the sand with his tail.” He also sent a paper to Cauchy revealing the double periodicity of elliptic functions—an impressive achievement—only to have Cauchy lose the paper. He took part in a rivalry of sorts with Jacobi in 1827 and 1828. Both men were working independently on the theory of elliptic functions—Abel learned of Jacobi’s work on the topic when reading a paper by Jacobi regarding transformations of elliptic integrals. Abel was furious, as he believed that Jacobi’s work was derived from his own, and set about rapidly writing papers on the theory of elliptic functions. While Abel did not live long enough to enjoy the fruit of his labors, the Paris Academy awarded Jacobi and Abel the Grand Prix for their work on elliptic function theory.
Historical events that marked Abel’s life.

At the end of the 18th century the political landscape of Europe was dominated by the Napoleonic wars. Norway—which was a part of Denmark at the time—attempted to avoid war, but was attacked by Britain in 1801, when the British destroyed the Danish fleet at Copenhagen. Surprisingly, this attack did not bring Denmark into war. The British again preemptively attacked in 1807, and this time captured the whole Danish fleet. When Denmark allied with the French, the British blockaded Norway. This crashed the Norwegian economy: it brought the importing of grain and the exporting of timber to a halt. Even after the war ended, the Norwegian economy was slow to recover, and poverty rates were greatly increased—Niels Abel was hardly the only Norwegian man in the 19th century to spend his whole life destitute.

Significant historical events around the world during Abel’s life

As mentioned previously, Western Europe’s political climate during the first thirteen years of Abel’s life was dominated by the Napoleonic Wars. Napoleon I attempted to expand the French Empire and faced resistance from Great Britain and Russia—while many other countries fought against the French, the British and the Russians had the greatest impact on the war. Napoleon was able to gain power in France by taking advantage of the extreme power vacuum left by the disastrous French Revolution of the 1790s, and the brutal subsequent reign of terror led by Robespierre. Napoleon was the world’s finest general, and the French quickly conquered territory throughout Europe. Napoleon had an almost perfect record in battle throughout the early years of power, and historians estimate that he won sixty battles while losing only seven. This statistic is made even more impressive by the fact that it includes the battles he lost in his final years of power. However, he made a grave mistake—which would be echoed in both World Wars—in deciding to invade Russia. His men were
unprepared for the harsh Russian winter, and the war was lost in 1814. Napoleon was imprisoned on Elba after the war, but he managed to escape and return to France. He then regained power and attempted to reignite the war. His luck finally ran out at the battle of Waterloo, and he would be exiled—this time permanently—to Saint Helena.

The Napoleonic wars had far reaching and unforeseen consequences. The Holy Roman Empire ceased to exist shortly after the war. Its territory spanned what would become Germany and Italy, and the Napoleonic wars laid the groundwork for their emergence as independent nations. Also, Spain lost its status as the world’s dominant nation and much of its worldwide prestige after being soundly beaten on the battlefield by the French. While focusing its military attention on Europe, Spain could not effectively suppress revolutions in Latin America, leading to reduction in the size of the Empire and the emergence of new independent Nations in the Americas. With Spain in decline and the French defeated, Great Britain became the world’s foremost power.
From the late 18th century to the mid 19th century the Industrial Revolution began to fundamentally change the world: it would upset the world’s balance of political power, lead to the development of entirely new economic systems, and eventually irrevocably change the everyday life of almost the entire world population. The Industrial Revolution began in Great Britain in the late 1760s and centered on using machines for production instead of manual labor. It began with the textile industry, and would eventually extend to the production of new chemicals, steel, and electricity. The development of the factory system would dramatically increase production efficiency and volume, and the invention of the steam engine would revolutionize transportation. Industrialization produced extremely lucrative businesses, and in the United States, created a financial environment that allowed men like JP Morgan, John D Rockefeller, and Andrew Carnegie to accrue wealth that rivaled that of entire nations. It had a profound impact on the international balance of power, as nations that were quick to industrialize—like the United States, Germany, and Japan—became world powers overnight while nations resistant to change quickly fell behind. The most striking example of this was China. China had dominated the far East since the days of the Qin dynasty in 500 BC, but in 1900 Japan—which had industrialized less than half a century prior—was one of the nations that had divided China into spheres of influence. Industrialization also encouraged urbanization, and began the decline of primarily agricultural economies. It led to an increased standard of living for the average person. It also led to the development of capitalism and communism—the economic and eventually ideological systems that would lay the groundwork for the cold war in the 20th century. Basically every aspect of modern life was made possible by industrialization.

Significant mathematical progress during Abel’s lifetime

Many groundbreaking mathematical developments were made in the early 19th century. The depiction of complex numbers was revolutionized in 1806, when Jean-Robert Argand developed Argand diagrams—which were a method of diagramming complex numbers geometrically. Carl Friedrich Gauss also lived during this time period, and he developed advanced mathematics in fields as diverse as geometry, number theory, calculus, algebra, and probability. In the nineteenth century non-euclidean forms of geometry were developed. Bernhard Riemann pioneered non-euclidian geometry through exploration of elliptic geometry and the zeta function. He also developed a new great unsolved problem in mathematics when he published his Riemann Hypothesis—the zeta function has zeros only at the negative even integers and the complex numbers with real part 1/2. He never produced a proof for his hypothesis, and even today it remains unproven (in fact, the proof of the Riemann Hypothesis played a role in the movies Proof and Fermat’s Room). In 1823 Charles Babbage designed one of the world’s first calculators—his “difference engine”—which was able to calculate logarithms and trigonometric functions. Babbage also designed a predecessor of modern computers which he dubbed the “analytic engine”.
Connections between history and the development of mathematics

Europe experienced an intellectual revolution at the end of the 18th century—which was sparked (or mirrored, depending on your perspective) by the political revolutions in America and France. In France, Napoleon helped encourage the development of mathematics with an emphasis on mathematics that could benefit society. This produced an intellectual environment favorable to the development of mathematics, and men like Joseph-Louis Lagrange, Pierre-Simon Laplace, and Adrien-Marie Legendre became mathematical giants. In Germany, an intellectual environment that favored mathematics also developed in the early 19th century—but its focus differed from that of France. In Germany, pure mathematics was favored over practical mathematics. Pure mathematics included the exploration and development of algebra, geometry, calculus, and number theory (although number theory is a very interesting case because it originally lacked practical purpose but would become extremely useful in computer science hundreds of years later) while practical mathematics dealt with the application of mathematics in computation, physics, the sciences, probability and statistics, and game theory (in the 20th century). The foremost German mathematician in this era—and of all time—was Carl Friedrich Gauss. The intellectual divide between those who pursue practical mathematics and those who pursue pure mathematics persists to this day. Practical mathematics is developed by and for engineers and computer scientists while other mathematicians still focus on mathematics for its own sake—the mathematical community recently celebrated the proof of Fermat’s Last Theorem, which had puzzled mathematicians and had been the most famous problem in mathematics for more than 400 years, even though it lacks extensive practical application.
Abel's life was not a happy one. He was born to poor alcoholic parents. His mathematical accomplishments were not financially profitable and did not raise him up from poverty. When he finally managed to get a job as a professor of mathematics in Berlin that would provide him with financial security, it was too late, as he died before receiving news of his appointment. Also, many of his accomplishments were not fully appreciated during his lifetime—as mentioned previously, one example of this is when Gauss refused to even look at his proof of the impossibility of finding a general solution in radicals to the quintic equation with arbitrary coefficients. Although he was engaged, the engagement dragged on for four years and he died before he could marry and start a family. It is a shame that his life ended at such a young age. It is especially sad to think about when you consider that his life was just about to improve dramatically. However, he did leave a lasting legacy. He was a pioneer in the field of mathematical analysis and in the study of functions. Many of his works were published posthumously by his mentor Holmboe and the Norwegian government in 1839, and then republished in 1881 by Ludwig Sylow and Sophus Lie. In mathematical writing groups that adhere to the commutative property are described as abelian, and the use of the adjective abelian is widespread. In his native Norway, his likeness has appeared on stamps, currency, and a statue in Oslo. The government has also established the Abel Prize—a prize equivalent to $600,000 in Norwegian kroner awarded to exceptional Norwegian mathematicians annually—in his honor. There is even a crater on the moon that bears his name.
**Famous Quotations** "With the exception of the geometrical series, there does not exist in all of mathematics a single infinite series the sum of which has been rigorously determined. In other words, the things which are the most important in mathematics are also those which have the least foundation."

"The mathematicians have been very much absorbed with finding the general solution of algebraic equations, and several of them have tried to prove the impossibility of it. However, if I am not mistaken, they have not as yet succeeded. I therefore dare hope that the mathematicians will receive this memoir with good will, for its purpose is to fill this gap in the theory of algebraic equations."

"The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever. By using them, one may draw any conclusion he pleases and that is why these series have produced so many fallacies and so many paradoxes."

"There are very few theorems in advanced analysis which have been demonstrated in a logically tenable manner. Everywhere one finds this miserable way of concluding from the special to the general and it is extremely peculiar that such a procedure has led to so few of the so-called paradoxes."

"A monument more lasting than bronze"–Adrien-Marie Legendre(quoting Horatius) describing Abel’s paper on the double periodicity of elliptic functions. Ironically the paper disappeared shortly after.

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