Nicolas Bourbaki

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Bourbaki's life

Nicolas Bourbaki is actually not a person, but instead is a pseudonym for a group of radical french mathematicians that worked to *"reinvigorate the mathematics of the 20th century"* to create updated, well written textbooks to replace older ones. This group was founded in 1935 by several exceptionally talented mathematicians. Their initial focus was on analysis, but their work quickly expanded to other areas of mathematics, including algebra, topology, and set theory. They wanted to write a modern text that encompassed the two main areas of mathematics, geometry and number theory, with a specialization in analysis. They believed that French mathematics was falling behind the rest of the world, and this motivated the group to greatly improve the current mathematical state of their country. The individuals were also frustrated with the dull and outdated analysis textbooks used at their respective universities.

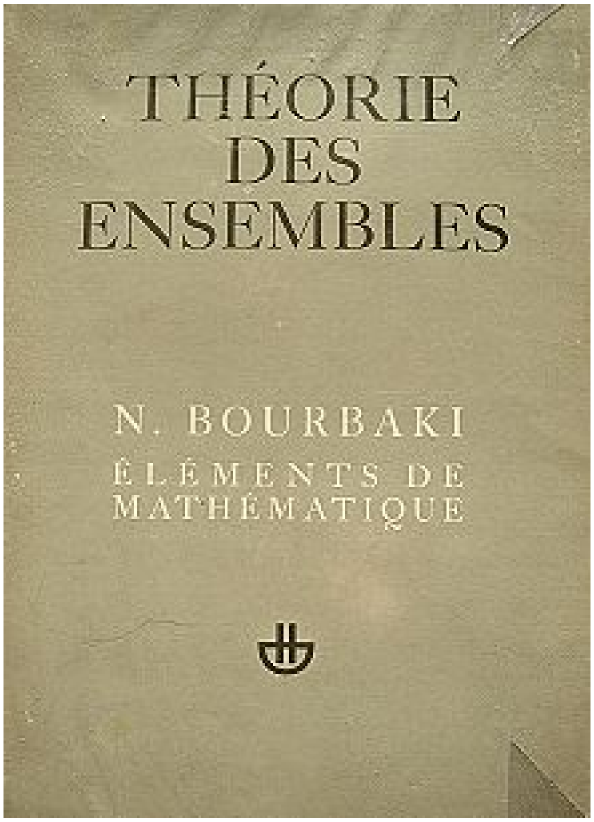
The members that began this group include Henri Cartan, Claude Chevalley, Jean Culomb, Jean Delsarte, Jean Dieudonne, Charles Ehresmann, Szolem Mandelbrojt, Rene de Possel, and Andre Weil. These members pledged to leave by age fifty, while many other youthful mathematicians continued to join and expand the influence of this group. A year after the group's founding in 1935, the memebers began to meet periodically three times a year in different locations around France for about one or two weeks. There, they decided that the group would under the name Nicolas Bourbaki, which comes from the French general Charles Soter Bourbaki who had fought in the Franco-Prussian war.

Most of the members were university students at the time, and thus wanted to have fun while they were in this group as well. When information about Nicolas Bourbaki being nothing more than a mere pseudonym for a group of university students is presented by R P Boas, the executive editor of *Mathematical Reviews*, the group challenged his authority saying Boas is just the same a pseudonym for the editors at *Mathematical Reviews*. All of the members spent time on their own research on different specialities apart from the work of Bourbaki. Their first work started as several drafts in an attempt to create a modern textbook for an analysis course, but the work and drafts continued to increase and eventually, this single course textbook became a multi volume series called *Elements de Mathematica*. This series was progressively published, as each additional volume was meant to cover a different subdiscipline of analysis.

Although none of the work done by Bourbaki was actual new information, his works did clarify and set a stronger foundation for the French mathematics. The works of Bourbaki were intended to develop rigorous mathematics that did not revolutionize the area but instead created a much more sophisticated presentation. Bourbaki's *Elements de Mathematica* created a standard for many areas of mathematics, particularly algebra, topology, and functional analysis. Several notations and symbols were also taken from this text and made standard in mathematics, including the empty set symbol, dangerous bend symbol, and the terms injective, surjective, and bijective.

Bourbaki's Mathematical Works

The Bourbaki group's most famous contribution, *Elements de Mathematica*, is a collection of ten books designed to modernize the French mathematics and put it on par with the rest of the world.



It's volumes include:

1. Set Theory
2. Algebra
3. Topology
4. Functions of One Real Variable
5. Topological Vector Spaces
6. Integration
7. Commutative Algebra
8. Lie Theory
9. Spectral Theory
10. Algebraic Theory

The first volume was published in 1939, which was named the *Fascicule de Resultats of Theorie des Ensembles*. Many of the volumes were completed by the 1980's and 90's, but to this day, the work as a whole remains incomplete. The first collection of these books was published in 1960. The collective project was the consequence of the laborious work of the Bourbaki group. During the group's scheduled meetings, they would create drafts of different chapters, and these drafts were then reviewed and revised by other members in the group. Once each draft was submitted and edited multiple times, it was ready for publication.

One idea that affected Bourbaki in his publication is the realization of how developed mathematics was becoming, and how different areas of mathematics were diverging more and more. Bourbaki had originally planned to present mathematics in its entirety within this single addition from its best point of view to create a sort of "encyclopedia of mathematics" of sorts, but these intentions were overly ambitious and Bourbaki had to settle for the sections listed above. However, the original work contained 25-30 volumes already on its own to begin with, and this was only a select group of topics, not including proofs. If the proofs had been a part of the text, not only would this have greatly extended the already long production for the work, but it also would have lengthened it by at least ten times its current size. This created a controversial question within the group; were these ideas separating in relation so much that the single mathematics was becoming several separate mathematics? In Bourbaki's work alone, it is apparent how intricate and distant the various subjects within mathematics has become. Structure was an important factor in this work, as the goal was to bring an up-to-date version of the areas mentioned.

A central portion of this work was to simplify the unnecessarily complicated language and terms commonly used in mathematics at the time. For example, the term "non-decreasing" is used to represent a series that may be increasing but may also just be constant. Bourbaki refused to use terms like these because the group felt they were redundant and insignificant and took away from the actual meaning of the work itself. So, instead of using "non-decreasing", the group would opt to say exactly what the series is doing, whether increasing, decreasing, or remaining constant, because "non-decreasing" is ambiguous. It is the simple things like these that eventually add up and are what made Bourbaki's *Elements* successful in reiterating previous works in a clearer, more effective manner.

Bourbaki's first book, Set Theory, was written as a foundation for the collection. It was intended to give insight into several mathematical tools and ideas, but instead became more of a biased opinion about several topics in math. Set theory is the area of mathematics of determined collections called sets made up of elements or members. These notions are built upon axioms, which are simple truths held to be self-evident. Bourbaki chose this to be the foundation for *Elements* because all theorems of mathematics can be evaluated through the axioms presented in number theory.

The group also covered the subject of analysis, which, simply put, is the area dealing with integration, differentiation, limits, and related series. Calculus is encompassed into this overarching subject, which also includes a form of abstract logic theory.

Another pivital subject covered by Bourbaki is that of topology, which is the *"area of mathematics concerned with the properties of space that are preserved under continuous deformations, such as bending and stretching, but not tearing or gluing"*. The studies presented in this subject have led to many scientific innovations, including the work on unusual states of matter that led three physicists to winning the nobel prize earlier this month on October 4th, 2016. Although although Bourbaki did not create any new mathematics, it is still interesting to see how a restructure of existing content can still benefit the work of others.

Collaboration with other scholars

The group initially used Bartel Leendert van der Waerden's *Treatise of Algebra* as an outline for their work. Waerden's work in itself paralleled Bourbaki's future project because this "Treatise of Algebra" was a collaboration with other mathematicians to produce a clear, concise final copy. Bourbaki particularly intended to emulate Waerden's *"very precise language and extremely tight organization of the development of ideas and ti the different parts of the work as a whole"*. This goes back to the group's goal to be exact and take away the ambiguity that made some of the current French works unclear.

Several of the individual mathematicians that were members of Bourbaki had significant individual merits in the mathematical areas.

Alexandar Grothendieck contributed significantly with his works in algebraic geometry, Cohomology theories, and Category theory. Alexander Grothendieck, although not an initial founder of Bourbaki, worked alongside several of the members that did found the group including Jean Dieudonne and Andre Weil. Grothendieck is considered to be by many to be the greatest mathematician of the 20th century, so although he did not contribute to the creation of Bourbaki, his cooperation with the group is definitely worth noting. Grothendieck worked in the areas of algebraic geometry, homological geometry, general logic, and categorical logic. His early work was in functional analysis. His most influencial work came in algebraic geometry in his *Tohoku Paper*, *"where he introduced abelian categories and applied their theory to show that sheaf cohomology can be defined as certain derived functors in this context."*. He modified already existing work in algebraic geometry and wrote it in a method that was *" at a higher level of abstraction than all prior versions."* Grothendieck also attempted a similar feat to the Bourbaki group by trying to encompass all of algebraic geometry into two main texts: *Elements de Geometrie Algebrique* and *Seminare de Geometrie Algebrique*

The other members that formed a part this group as mentioned previously include founding members Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Alexandre Eugene Dieudonne, and Andre Weil. Cartan was a renowned french mathematician and one of the most active participants of the Bourbaki group. He particularly focuses on algebraic topology, which earned him the Pagels Award from the New York Academy of Sciences. He also had the Cartan model named after him. One of his most famous works was with the category theory.

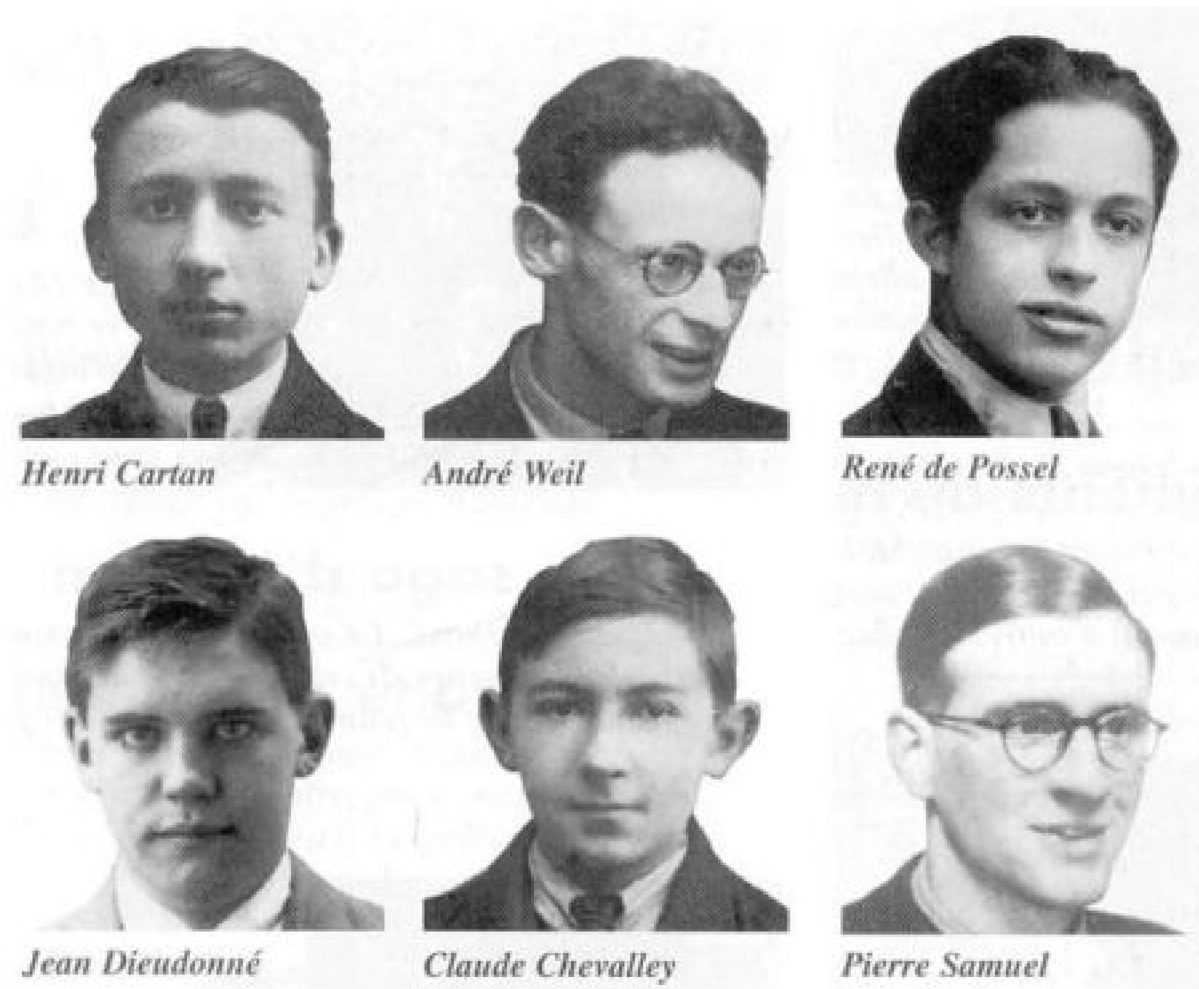
Chevalley was similarly as gifted, again specializing in number theory, particular famous for his developement of class field theory. He wrote a three volume series on Lie groups, which, simply put, is a theory of continuous symmetry of mathematical objects and structures. They were a key tool in both contemporary mathematics and theoretical physics. These Lie groups became known as Chevalley groups which form 9 out of the 18 finite simple groups.

Historical events that marked Bourbaki's life.

The first world war played a significant role in the formation and "life" of the Bourbaki group. During the war, the French government decided that it would be of more benefit to the nation if all the nation's young men served in the army, particularly infantry combat. This meant that despite an individual's occupation, they had to serve in the front lines for their nation. Many of the nation's greatest future minds were sent to serve in the war. Although this sign of patriotism and dedication can only be respected, it is a decision that significantly deteriorated the French science and mathematical communities. The Germans took a different approach of sending the native scientists and mathematicians to work on technological innovations for their country to be used in war instead of sending them into direct combat, and this was a more successful attempt which propelled German industry even after their eventual defeat in the war. France, however, was not as fortunate, as a considerable portion of the nation's great minds had died contributing to war amongst the two-thirds of the total French ranks that had been lost in it. As a result, there was left a gap in the nation's community of mathematicians and scientists that caused the technical studies of the nation to lag behind that of the other European nations. The period of the war and shortly after, the French mathematics stalled while other nations continued to advance.

However, there was an instance following the war that managed to keep the French mathematics alive. A French professor by the name of Jacques Hadamard who taught mathematics at the *College de France* offered to give a seminar on analysis on current mathematical work. This was very beneficial to the progress of French mathematics because apart from discussing contemproary mathematics that not many were familiar with, this seminar brought over many brilliant mathematicians from all over Europe to bring their influence over to French territory. This was the opportunity France needed to get back on track with the rest of the world regarding mathematics. This was a great relief to the French because they carried a history of great mathematicians such as Peirre de Fermat and Henri Poincare to name a few. They did not want this legacy of influential mathematicians to end there, so this seminar was a key element in the French regaining their title.

The actual group was formed when Andre Weil and Henri Cartan, both french mathematicians attending the I'Ecoli Normale, one of the most pestigious universities in the nation at the time alongsode Polytechnic. There, while the two were studying differential and integral calculus, both students were frustrated with the outdated and convaluted textbooks that they were given for the class. They felt that the recommended text, Goursat's *Traite d' Analysis* was inadequete and had room for improvement. For this reason, the two planned to write an entirely new textbook alongside their friends who were attending the periodic mathematical seminars. The two consistently visited Paris were they would attend the seminars and there they also discussed their plan with other mathematicians. From that point, Henri Cartan and Andre Weil planned to work alongside other mathematicians including Jean Delsarte, Jean Dieudonne, and Rene de Possel, and thus, Nicolas Bourbaki was born.



Henri Cartan — André Weil — René de Possel

Jean Dieudonné — Claude Chevalley — Pierre Samuel

Significant historical events around the world during Bourbaki's life

After the formation of the group, Bourbaki's progress was halted during the second world war. Not much had been done in the events leading up to the war; books 1, 2 and 3 of Bourbaki's *Elements* had been temporarily halted for the period of the war. However, the war this time proved to be much less detrimental to the French than the first world war as many of the mathematicians remained to continue contributing to the group. There was an additional break of 5 years after the war that the group did not publicize any work. Also, after the war, there was an influx of new mathematicians that were recruited to join the efforts of Bourbaki. These were considered "second generation" members that were necessary to continue keeping the group youthful and open minded to contemporary ideas.

Significant mathematical progress during Bourbaki's lifetime

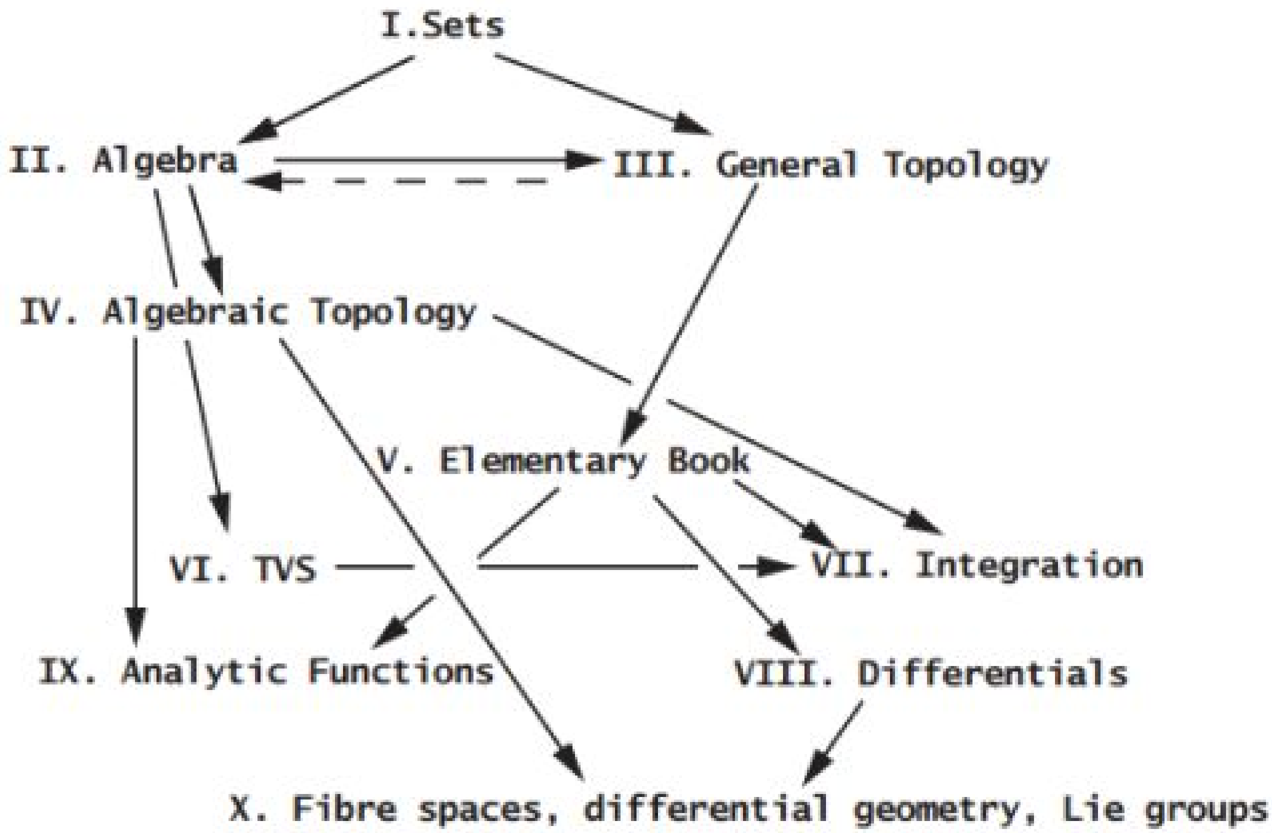
As stated before, Bourbaki did not specifically develop any new mathematics in the group's most famous work, *Elements de Mathematica*, but instead created a new standard for the manner in which mathematical works were written. The work was written to show *"the best point of view"* possible of each topic covered. They wanted to both clearly and sophisticatedly describe these topics to create a new standard for showing works for the mathematical community.

The two main features they wanted to maintain as a group were to:

1. Set forth mathematical theories that were already completed in a comprehensible and understandable way. The group focused on mathematical structure; their works contain no original imformation, as their purpose was not to innovate, but to provide clarity for what already existed. In the words of Jean A. Dieudonne, a brilliant french mathematician that contributed to the founding of Bourbaki, *"What Bourbaki has done is to define and generalize an idea which already was widespread for a long time ... There is no question of Bourbaki's containing anything original, ... if a theorem is in Bourbaki, it was proven 2, 20, or 200 years ago."* That is not to say the work of this group has not benefited the mathematical community. What Bourbaki has done is to create a standard for existing mathematics, and this is something that can be of great benefit to future generations of mathematicians and the mathematical community as a whole. Bourbaki points out the distinction between discovering a mathematical work and actually understanding it. Although many breakthroughs can come through the discovering of new mathematical concepts and ideas, this does not mean it is imeadiatley an innovation worth noting. Over time, others will try to understand these innovations because for the most part, these intricate proofs in their original form ar far from their best representation. It may take years, decades, even centuries for some of these ideas to be perfected and fully understood. That was the primary goal of Bourbaki- to take these already discovered concepts, whether they were recently discovered or been existent for a significant amount of time, and then write and rewrite them until a final, cohesive, clear, concise copy can be produced. This process would take the mathematicians that formed Bourbaki several years to just finalize one volume of one subject matter in the list of subjects that they planned to cover. It is ironic that I say the word "concise", because their single volume finalized copy of their first book on Set Theory spans over 400 pages and took over 30 years to finish as this was published for print in 1970. Many of the later works covered by Bourbaki are still incomplete and are still being updated with new drafts, giving a glimpse of how intricate the language of math actually is.

2. Categorize mathematics into a series of foundational topics that can be followed into every area of mathematics. This meant deciding what branches of mathematics were important enough to be included in their works. Perhaps this was even more important than actually rewriting the works because even if none of the mathematical concepts were updated, this defined what areas of mathematics were the "roots" which led to further development. This is not to say certain areas of mathematics are "good" and others are "bad"; Bourbaki was only separating the different subjects into those that could lead to further exploration and application and those that did not. Math can be considered an art, similar to that of traditional literature because mathematics in itself is a language. Those who write literature may do it for a particular purpose, or they may do it just for the sake of writing. This is because to a true writer, it does not matter to them if a certain piece of writing was written as a social criticism or just for the aesthetics of writing. They appreciate good writing all the same. Similarly, I believe mathematics to be an art just the same. Some of it may be applied in sciences that lead to the development of society, but others are just there for appreciation of the work. Going back to

the purpose of Bourbaki, by differentiating subjects by "importance", they were not criticizing certain areas and praising others, just categorizing based on what led to future development and what did not.



These principles are what has allowed Bourbaki's books to become standard in certain fields of mathematics. They mainly influenced graduate level mathematics, and were used as references for the teachings of "pure mathematics". This influence was strongest during the 50's and 60's when there existed few other graduate texts in "pure mathematics".

Connections between history and the development of mathematics

Leading into the 20th century, France was home to some of the greatest mathematical minds to exist amongst more "modern" mathematics. Some of these famous mathematicians include Pierre de Fermat, Evariste Galois and Henri Poincare. The french have excelled at mathematics for for centuries leading up to modern day. However, there were some geopolitical events that did interrupt french mathematical prestige. Political trunmoil at the end of the 18th century did cause a roadblock in the development of the french technical areas, but the outcomes of the revolution outweighed the time that was spent under political turmoil. The french higher education system was reformed and improved; the addition of the new school, Ecole Polytechnique, allowed for the future great minds within France through the help of talented professors such as Joseph-Louis Lagrange ment to nurture other students that exemplified similar potential in a cyclical process. Although the new focus for french higher education was oriented towards engineering, mathematics is a key part to engineering and benefited greatly from this new education system as well.

As stated earlier, it was the decline in the technical areas of the French nation following World War I that caused a need for the French to improve these areas. In mathematics, the need was just as great, so when

Jacques Hadamard offered to give a seminar on analysis, this revived the dying French mathematics. When Hadamard retired from these seminars, Gaston Julia offered to lead the group's gatherings to continue the spread of contemporary mathematics. From there, the idea proposed by Andre Wiel and Henri Cartan created this iconic group that continues to exist today.

Remarks

An idea that stood out for me while researching these mathematicians and their works is the idea that despite the significance of some of these people's works, they do not seem to get enough credit for their efforts when compared to the works of other technical areas. This relates back to the idea of mathematical work that is practical versus mathematical work that is just for appreciation. When a mathematical discovery is made, the mathematical community does not bother to take in consideration its application. The majority of the time, whether or not a mathematical piece is useful will not be known until later in the future. For example, the Prime Number Theorem was proven in 189, but at the moment, there was no immediate application for the discovery. It is not until the second world war when the first computers were being used that this became useful in the area of cryptography. For this reason, mathematicians are sometimes not accredited for their work until after their death.

There is also two main viewpoints held about Bourbaki and his works. Some praise the group for successfully categorizing the foundational mathematical areas and creating a new standard for future generations to benefit from. Others, however, criticize Bourbaki's limited view of mathematics and feel that by excluding many other areas of study, the group essentially took away from mathematics by only focusing on a select few areas of study. Both are valid viewpoints, and each side has a different approach to mathematics and what its true meaning is.

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