Leonhard Euler was one of the greatest mathematician and physicist of all time for his many contributions to mathematics. His works have inspired and are the foundation for modern mathematics. Euler was born in Basel, Switzerland on April 15, 1707 AD by Paul Euler and Marguerite Brucker. He is the oldest of five children. Once, Euler was born his family moved from Basel to Riehen, where most of his childhood took place. From a very young age Euler had a niche for math because his father taught him the subject. At the age of thirteen he was sent to live with his grandmother, where he attended the University of Basel to receive his Master of Philosophy in 1723. While he attended the University of Basel, he studied Greek in Hebrew to satisfy his father. His father wanted to prepare him for a career in the field of theology in order to become a pastor, but his friend Johann Bernoulli convinced Euler’s father to allow his son to pursue a career in mathematics. Bernoulli saw the potential in Euler after giving him lessons. Euler received a position at the Academy at Saint Petersburg as a professor from his friend, Daniel Bernoulli. He rose through the ranks very quickly. Once Daniel Bernoulli decided to leave his position as the director of the mathematical department, Euler was promoted. While in Russia, Euler was greeted/introduced to Christian Goldbach, who sparked Euler’s interest in number theory. Euler was a man of many talents because in Russia he was learning Russian, executed studies on navigation and ship design, cartography, and an examiner for the military cadet corps. He even began plans to for a basic water turbine in order to replace them with a single cylinder steam engine with a propeller. On January 7, 1734 Euler wedded Katharina Gsell, a daughter of a painter from the Academy Gymnasium. Katharina and Euler had thirteen children, but only five of them made it to adulthood. In Euler’s autobiographical writing he claimed that some of his greatest mathematical works came from when he was holding a baby and children were running around his feet. Euler was the first to comprehensively explain Newtonian Dynamics through mathematical analysis.
Euler left Russia because of the political mayhem that made him fear his safety; he went to Berlin, where he accepted the position at Berlin Academy of Science from Frederick the Great of Prussia. During his time period in Berlin, he wrote the two most distinguished, which were the *Introductio in Analysin Infinitorum* (on functions) and *Institutiones calculi differentialis* (on differential calculus). During the twenty five years that Euler was in Berlin, he was the most productive and successful. He spent his time doing a variety of things that were also not mathematical such as serve on the Library and Publications committee of the Berlin academy. Also, he dabbled in politics and became a government advisor on insurance, annuities, pensions, and artillery. Euler was also asked to tutor princess of Anhalt Dessau, Friederike Charlotte of Brandenburg-Schwedt. He sent her over two hundred letters in the 1760s that discussed various subjects in mathematics in layman terms. These letters later become a best-selling volume called *Letters of Euler on different Subjects in Natural Philosophy Addressed to a German Princess*. He provided to the Academy’s standing. However, he could not avoid the anger of Frederick, the king. He was not a skilled debater, and was a religious man. Voltaire and Euler were complete opposites; therefore, Frederick was pleased with Voltaire. Euler faced number of tragedies during his lifetime. His health was deteriorating. He began with a fever that nearly caused his death, but it transformed into the deterioration of his right eye. Throughout his stay in Germany, his vision continued to deteriorate. Frederick called him cyclops because he was only able to see through one eye. In 1766, it was discovered that Euler had developed a cataract in his left eye. Not long after the discovery he was identified as blind. This limitation did not stop Euler from continuing his mathematical endeavors. First signs of the eye problem began after the death of Catherine I, and backlash from the foreigners who dominated the academy, which occurred in 1738. He still kept his connection with the St. Petersburg Academy, especially since they were still funding him.

An interesting fact about Leonhard Euler is that he had an extraordinary memory and concentration. He was able to recite *Aeneid*, which was an epic poem, off memorization. Euler did not get distracted because he did majority of his work surrounded by children at his feet. Also, he was capable of doing mental math, and that became very useful for when his vision failed him. Another sad occurrence that happened to him was when his wife died, but he later married his half sister (He must have loved his wife’s face so much he had to get a close second). Even with the death of his wife he continue his work at such a marvelous speed. His vision was completely lost so the remaining of his works were written in scribes.

### Euler’s Mathematical Works

<table>
<thead>
<tr>
<th>Mathematics notation created or made popular by Leonhard Euler:</th>
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<tbody>
<tr>
<td>$e$</td>
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<tr>
<td>$i$</td>
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<tr>
<td>$f(x)$</td>
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<td>$\Sigma$</td>
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<td>$a,b,c$</td>
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<tr>
<td>$x,y,z$</td>
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<tr>
<td>$\sin,\cos,\tan$</td>
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<td>$\pi$</td>
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2
Leonhard Euler is the father of modern mathematics, there is probably not a single field of mathematics that Euler did not influence. Euler’s interests incorporates almost all categories in the mathematical field: calculus, geometry, optics, astronomy, mechanics, weights, algebra, number theory, cartography, and the theory of music. His contributions are being used in modern mathematics, especially since he created many of the fundamentals to compute certain math problems and equations. Euler has created a number of theorems. The image above list the notation that Euler created or has popularized, and this allowed mathematics to be internationalize. Euler uses some of these notation to create one his most famous mathematical equation of all time

\[ e^{i\pi} = -1 \]

This equation derives from a more general equation, which includes the notation of the trigonometric functions:

\[ e^{ix} = \cos(x) + i\sin(x) \]

This equation expresses "exponential, imaginary growth traces out a circle". And this path is the same as moving in a circle using sine and cosine in the imaginary plane." It is a form of complex analysis that is a underlying relationship between trigonometric functions and complex exponential function. Euler was also a forefront for calculus and known for analysis, especially from his development of the power series. His power series also incorporated his famous equation that was stated above. He also discovered the power series for the trigonometric functions, and this allowed him to solve the Basel problem in 1735. The expansion below following is Euler expansion for \( e^x \) :

\[ e^x = \sum_{n=0}^{\infty} \frac{1}{n!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} \]

This allowed him to solve the Basel problem which was

\[ B = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots + \frac{1}{k^2}. \]

Daniel Bernoulli first estimated that the answer to the problem was \( \frac{8}{9} \), but it turned out to be wrong. Euler calculated the true solution to the Basel problem was \( \pi^2 / 6 \). He also was able to show that the infinitie series was equivalent to the infinite product of prime numbers, which will later help Riemann with his investigations. Euler found the solution to the Königsberg Bridges problem. The solution to this problem developed the field of mathematics known as Graph Theory. Graph Theory is "the study of graphs, which are mathematical structures use to model pairwise relations between objects. Euler’s polyhedral formula was created when he first began to take a polyhedral apart piece by piece, this he how he gained a fair view of polyhedral. This allowed him to solve the bridge problem. The equation that Euler created was \( V + F - E = 2 \).
The following image shows how he relates the numbers of edges and vertices, and faces of polyhedral in the solution.

**Collaboration with other scholars**

Compared to many mathematicians, Euler did not collaborate with many other individuals. The only ones that were evident was his collaboration with Daniel Bernoulli and Christian Goldback. There correspondence with one another lasted for about 35 years. During this time period Euler incorporated mathematics into music. This aspect of his work was not as accepted especially, by musicians because they believe that there was too much mathematics rather than music. Through their letters to each other they discussed Fermat’s Last Theorem that assisted them to gain a understanding create his Goldbach last conjecture. It has been widely stated that Johann Bernoulli was the reason why Euler is one of the greatest mathematicians of all time. He took him under his wing, and took him away from becoming a pastor.

**Historical events that marked Euler’s life.**

The most important event or era that took place during the life span of Euler was the Enlightenment era. The enlightenment era is defined as “a European intellectual movement of the late 17th and 18th centuries emphasizing a reason and individualism rather than tradition. It was heavily influenced by 17th century philosophers such as Descartes, Locke, Newton, and its prominent exponents include Kant, Goethe, Voltaire, Rousseau, and Adam Smith.” This benefited Euler because people were not conservative to different ideas, and allowed him to explore different ideas in mathematics as well as anatomy, astronomy, and many other fields.

Euler during his lifetime resided in two locations, St. Petersburg and Berlin. In the 1700’s there were many historical events that took place. He began in Switzerland where he completed most of his childhood. The reason for his departure was because his friends Daniel Bernoulli were leaving. He was hesitant about leaving for St. Petersburg because he was waiting to see if he was going to receive the position at University of Basel. For the application process he wrote a paper *Dissertatio physica sono* (Physical Dertation of Sound). His application was denied, and therefore made his decision to go to Berlin much quicker. The founding of the St. Petersburg was founded by Gottfried Wilhelm von Leibniz and Emperor Peter I of Russia. Peter began to write to Leibniz about the
condition of Russian science. Through the inquiries Leibniz wanted to advertise scientific inquiry and education. The beginning of the science reform movement in Russia was a letter sent in 1716 from Leibniz listing his opinions and contributions for the educational reform in the Russian Empire. Then the committee/advisors began to persuade foreign scientists to become a part of the new academy. New members started to arrive in 1725.

Some of the members originally at the Academy were Daniel and Nicholas Bernoulli, Christian Goldbach, Johann Duvernoy, and Christian Gross. Tragically Peter died on January 28, 1725, and was not able to see his academy flourish. Peter’s widow, Catherine I was appointed to finish the carrying out and the formation of the Academy. When Euler arrived at the Academy he began in the field of physiology, a field where he had little experience. Not long after he was moved to a different area of study. He became a Professor of Physics in 1731 and a Professor of Mathematics in 1733. He also was given the role as the Academy’s Geography and Cartography department. The political atmosphere in Russia was worsening, and this was the main reason Euler left St. Petersburg to go to Berlin. Not long after Euler’s arrival to St. Petersburg, Catherine I dies, and was succeeded by Peter II. Peter the II was the grandson of Peter I, and he is only twelve years old. The problem is that the young new emperor falls sickly often, and was often influenced by his conventional advisors. He was convinced to move the imperial capital to Moscow, and his reign cause the decline of the Russian science movement and the Academy. Peter died, and the grand-niece of Peter I, Anna Ivanovna was the new empress. The public disliked her great, especially since majority of her advisors were German. Over time, the Russian public became xenophobic and antagonistic towards Anna and her German advisors. This reign just caused stagnation, and after the death of Anna in 1740 things just began to get even worse. Anna’s grand nephew, Ivan VI was appointed, but was only an infant; therefore his mother had to step in as regent because Ivan was so young. 1740-1741 just caused Russia to go down a political spiral causing turmoil across the country, which causes concerns for Euler and his family. They end up deciding to travel to Berlin because Russia was no longer safe for him and his family.

Euler and his family went to Berlin where continued to build on his mathematical and other achievements. He accepted a position at the Societas. A little background history on the Berlin Academy was that it was the idea or initiative of Gottfried Willhelm von Leibniz beginning in the 1690s. He petitioned to establish a Germany academy of sciences. After 20 years of dedication, the Academy was established in 1710, and it was called Societas Regia Scientiarum. Leibniz left Berlin, and the Academy was placed under the rule of Frederick William I. The Academy went through a decline because Frederick did not find any value in the Academy. The Academy was still able
to write a semi regulated journal, *Miscellanea Berolinesia*. The Academy went through a revival once Frederick II on the Prussian throne. Over four years, Frederick II was working relentlessly hard to make reforms and policy in the fields of science, military, and etc. During Euler time period in Berlin he publishes *Introductio in analysin infinitorum* in 1748, and *Institutiones Calculi Differentialis* in 1755.

Euler was unhappy with his situation in Berlin. He considers the offer that Catherine (the empress of Russia at that time) persuades him, which was to take a position at the St. Petersburg Academy. He had to also put in the consideration that his relationship with Frederick was soiled. Catherine incorporated many benefits such as providing him a fully furnished house for him and his many dependents, and giving him one of her cooks to run the kitchen. She treated Euler, as if he was royalty. While was at St. Petersburg, for the second time, he made progress on removing Catherine in order to bring Caspar Wolff, Samuel Gmelin, and Simon Pallas to join the Academy’s faculty. The changes were unsuccessful, and did not improve the conflict between the German and Russian faculty. The second time around in St. Petersburg, Euler made more scholastical progress than last time. He wrote almost half of his 856 catalogued works. The Academy published it entirely by the mid 1800s.

**Significant historical events around the world during Euler’s life**

[Image of world map]

When Euler was born, 1707, this is when the United Kingdom of Great Britain formed, which is England, Wales, and Scotland (joined by parliamentary Act of Union). The Tsar of Russia in 1740 employs Captain Vitus Jonassen Bering, who captained the First Kamchatka Expedition. Captain Bering discovers Alaska. As this event was occurring Frederick II was crowned the king of Prussia. In 1746, the British defeat the Scottish under Stuart Pretender Prince Charles at Culloden Moor, and the last battle was fought on British soil. France begins the publication of the Encyclopédie, which was known to be the bible of the Enlightenment. In 1756 the Seven Years’ War where Britain and Prussia defeat France, Spain, Austria, and Russia. France lose their North American colonies. Spain has to give Florida to Britain in exchange for Cuba. In 1757 The British empire begins in India as Robert Clive, a British commander, defeats Nawab of Benegal at Plassey.

**Significant mathematical progress during the X’s lifetime**
During the 1700s there were many mathematical developments, many of them were contributed to by Euler. One of the influences in Euler’s life, Johann Bernoulli, declared in 1717 that ”the principle of virtual displacement is applicable to all cases of equilibrium.” Johann brother Jacob calculus of variations is published after he dies in 1718. Another important contribution to calculus was given by Brook Taylor, which was his publication of *Methodus incrementorum directa et inversa.* In his publication it discussed answers to differential equations, change of variables formular, and relating a derivative of function to the derivative of the inverse function. Brook Taylor publishes the *New Principles of Linear Perspective* which gives the general anecdote of vanishing points. In 1722 Cotes unfinished work was as *Harmonia Mensuratum.* Discussed in this publication was the integration of rational functions, and contains applications that coincide with logarithmic and circular functions. In 1724, the Academy of sciences is founded in St. Petersburg, which is where Euler is appointed to work in 1727. During this time he introduced the symbol e for the base of natural logarithms in a manuscript called *Meditation upon Experiments made recently on firing a Cannon,* which was not published until 1862. In 1726 Grandi gives a definition of geometrical curves, which look similar to petals and flower leaves; this definition was published in *Flora Geometrica.* De Moivre gives theorems on his trigonometric interpretation of complex numbers, as well as giving Stirling’s formula (discusses series and summations). In 1738, Euler’s friend, Daniel Bernoulli publishes Hydrodynamics. In his publication it gives the first correct interpretation of flowing water from a hole in a container, discuss pumps, and other machines to raise water. In 1730 D’Alembert publishes his Memoir on Integral Calculus. D’Alembert publishes *Treatise on Dynamic.* In his work he discusses his principle on the ”internal actions and recations of a system of rigid bodies are in equilibrium.” He further develops the theory of complex numbers, so he can prove the fundamental theorem of algebra. Euler publishes *Analysis Infinitorum* (Analysis of the Infinite). This was an introduction to mathematical analysis. Mathematical Analysis is the study of functions. His famous formula appears in the text.

**Euler in Modern Mathematics**

Euler is credited for being consistent with using lowercase letters represent the sides of a triangle, and uppercase letters represent the opposite angles. Euler was the first to establish the relationship of sets using circles, and they are identified as Venn Diagrams. Many of his equations, theorems, and other discoveries are named after him. When it was Euler’s 306th birthday, Google made a doodle with Euler reciting *Aenid* in the background. Euler’s contributions are remembered, and still being put to good use to solve and interpret new information. Euler’s work is inspiration for modern day mathematicians.
Remarks

Leonhard Euler was a remarkable individual who continued to strive even through his hardships. Even though he went blind, he continued to strive to make his footprint on Earth a lasting one. He has made a large impact for the future of mathematics. He has been constantly discussed in mathematics courses and the history of mathematics for his numerous contributions that assist modern mathematicians in their findings. Personally, I want to strive to be like Euler because he was a man of persistency and dedication. I have never heard of someone work at normal pace until their death. It would have been expected that he took a break or vacation to get his mind clear of all stress and tragedies that have occurred through out his life. He didn’t even let his lost of vision get in his way or the people who were against him.

Many people have written about Euler, and discuss his ethics and mannerism. Just by looking through his works one can just imagine what was going through his mind as he was discovering and answering different problems and equations. It is interesting because he used his loved of mathematics in different areas as well.
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